Th 2 ABSOLUTE MEASUREMENTS OF ELECTRICAL QUANTITIES

SOLUTION

1. After some time *t*, the normal to the coil plane makes an angle ωt with the magnetic field $\vec{B}_0 = B_0 \vec{i}$. Then, the magnetic flux through the coil is

$$\phi = N \; \vec{B}_0 \cdot \vec{S}$$

where the vector surface $\vec{S}$ is given by $\vec{S} = \pi a^2 \left(\cos \omega t \, \vec{i} + \sin \omega t \, \vec{j} \right)$

Therefore $\phi = N \pi a^2 B_0 \cos \omega t$

The induced electromotive force is

$$\varepsilon = -\frac{d\phi}{dt} \qquad \Rightarrow \qquad \boxed{\varepsilon = N \pi a^2 B_0 \, \omega \sin \omega t}$$

The instantaneous power is $P = \varepsilon^2 / \mathrm{R}$, therefore

$$\boxed{\left\langle P \right\rangle = \frac{\left(N \, \pi \, a^2 B_0 \omega \right)^2}{2R}}$$

where we used $< \sin^2 \omega \, t >= \frac{1}{T} \int_0^T \sin^2 \omega t \, dt = \frac{1}{2}$

2. The total field at the center the coil at the instant *t* is

$$\vec{B}_t = \vec{B}_0 + \vec{B}_i$$

where $\vec{B}_i$ is the magnetic field due to the induced current $\vec{B}_i = B_i \left(\cos \omega t \, \vec{i} + \sin \omega t \, \vec{j} \right)$

with $\qquad B_i = \frac{\mu_0 N I}{2a} \qquad$ and $\qquad I = \varepsilon / R$

Therefore $\qquad B_i = \frac{\mu_0 N^2 \pi a B_0 \, \omega}{2R} \sin \omega t$

The mean values of its components are

$$\left\langle B_{ix} \right\rangle = \frac{\mu_0 N^2 \pi a B_0 \, \omega}{2R} \left\langle \sin \omega t \cos \omega t \right\rangle = 0$$

$$\left\langle B_{iy} \right\rangle = \frac{\mu_0 N^2 \pi a B_0 \, \omega}{2R} \left\langle \sin^2 \omega t \right\rangle = \frac{\mu_0 N^2 \pi a B_0 \, \omega}{4R}$$

And the mean value of the total magnetic field is

$$\left\langle \vec{B}_t \right\rangle = B_0 \, \vec{i} + \frac{\mu_0 N^2 \pi a B_0 \, \omega}{4R} \vec{j}$$

The needle orients along the mean field, therefore

$$\tan \theta = \frac{\mu_0 N^2 \pi a \omega}{4R}$$

Finally, the resistance of the coil measured by this procedure, in terms of θ, is

$$\boxed{R = \frac{\mu_0 N^2 \pi a \omega}{4\tan\theta}}$$

3. The force on a unit positive charge in a disk is radial and its modulus is

$$\left|\vec{v} \times \vec{B}\right| = v\,B = \omega r\,B$$

where B is the magnetic field at the center of the coil

$$B = N\frac{\mu_0 I}{2a}$$

Then, the electromotive force (e.m.f.) induced on each disk by the magnetic field B is

$$\varepsilon_D = \varepsilon_{D'} = B\omega\int_0^b r\,dr = \frac{1}{2}B\omega b^2$$

Finally, the induced e.m.f. between 1 and 4 is $\varepsilon = \varepsilon_D + \varepsilon_{D'}$

$$\boxed{\varepsilon = N\frac{\mu_0 b^2 \omega I}{2a}}$$

4. When the reading of G vanishes, $I_G = 0$ and Kirchoff laws give an immediate answer. Then we have

$$\varepsilon = I\,R \qquad \Rightarrow \qquad \boxed{R = N\frac{\mu_0 b^2 \omega}{2a}}$$

5. The force per unit length f between two indefinite parallel straight wires separated by a distance h is.

$$f = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{h}$$

for $I_1 = I_2 = I$ and length $2\pi a$, the force F induced on C_2 by the neighbor coils C_1 is

$$\boxed{F = \frac{\mu_0 a}{h} I^2}$$

6. In equilibrium

$$m g x = 4 F d$$

Then

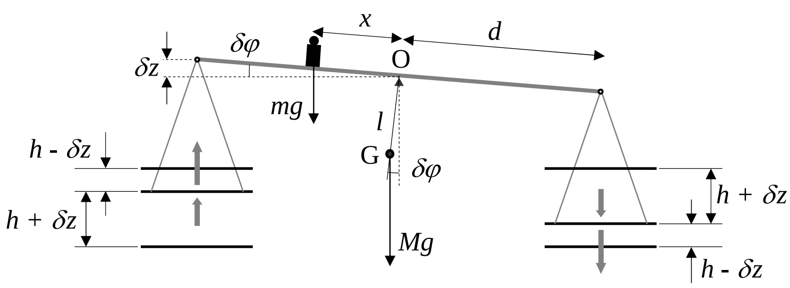
$$m g x = \frac{4\mu_0 a d}{h} I^2 \qquad (1)$$

so that

$$\boxed{I = \left(\frac{m g h x}{4\mu_0 a d}\right)^{1/2}}$$

7. The balance comes back towards the equilibrium position for a little angular deviation $\delta\varphi$ if the gravity torques with respect to the fulcrum O are greater than the magnetic torques.

$$Mgl\sin\delta\varphi + mgx\cos\delta\varphi > 2\mu_0 aI^2\left(\frac{1}{h-\delta z}+\frac{1}{h+\delta z}\right)d\cos\delta\varphi$$



Therefore, using the suggested approximation

$$Mgl\sin\delta\varphi + mgx\cos\delta\varphi > \frac{4\mu_0 adI^2}{h}\left(1+\frac{\delta z^2}{h^2}\right)\cos\delta\varphi$$

Taking into account the equilibrium condition (1), one obtains

$$M\,g\,l\,sin\,\delta\varphi > m\,g\,x\frac{\delta z^2}{h^2}cos\,\delta\varphi$$

Finally, for $\tan\delta\varphi \approx \sin\delta\varphi = \dfrac{\delta z}{d}$

$$\delta z < \frac{Mlh^2}{mxd} \quad\Rightarrow\quad \boxed{\delta z_{\max} = \frac{Mlh^2}{mxd}}$$

Th 2 ANSWER SHEET

Question	Basic formulas and ideas used	Analytical results	Marking guideline
1	$\Phi = N\,\vec{B}_0 \cdot \vec{S}$ $\varepsilon = -\dfrac{d\Phi}{dt}$ $P = \dfrac{\varepsilon^2}{R}$	$\varepsilon = N\pi a^2 B_0 \omega \sin\omega t$ $\langle P\rangle = \dfrac{(N\pi a^2 B_0\omega)^2}{2R}$	0.5 1.0
2	$\vec{B} = \vec{B}_0 + \vec{B}_i$ $B_i = \dfrac{\mu_0 N}{2a} I$ $\tan\theta = \dfrac{\langle B_y\rangle}{\langle B_x\rangle}$	$R = \dfrac{\mu_0 N^2 \pi a\omega}{4\tan\theta}$	2.0
3	$\vec{E} = \vec{v}\times\vec{B}$ $v = \omega r$ $B = N\dfrac{\mu_0 I}{2a}$ $\varepsilon = \int_0^b \vec{E}\,d\vec{r}$	$\varepsilon = N\dfrac{\mu_0 b^2 \omega I}{2a}$	2.0
4	$\varepsilon = RI$	$R = N\dfrac{\mu_0 b^2\omega}{2a}$	0,5
5	$f = \dfrac{\mu_0}{2\pi}\dfrac{I I'}{h}$	$F = \dfrac{\mu_0\,a}{h} I^2$	1.0
6	$mgx = 4Fd$	$I = \left(\dfrac{mghx}{4\mu_0\,ad}\right)^{1/2}$	1.0
7	$\Gamma_{grav} > \Gamma_{mag}$	$\delta z_{\max} = \dfrac{Mlh^2}{mxd}$	2.0