33rd
INTERNATIONAL
PHYSICS OLYMPIAD
2002
BALI - INDONESIA

# THEORETICAL COMPETITION

Tuesday, July 23rd, 2002

## Solution I: Ground-Penetrating Radar

**1.** Speed of radar signal in the material $v_m$:

$$\omega t - \beta z = \text{ constant } \rightarrow \beta z = \text{-constant } + \omega t \quad \text{(0.2 pts)}$$

$$v_m = \frac{\omega}{\beta}$$

$$v_m = \frac{1}{\omega\left\{\frac{\mu\varepsilon}{2}\left[(1+\frac{\sigma^2}{\varepsilon^2\omega^2})^{1/2}+1\right]\right\}^{1/2}} \quad \text{(0.4 pts)}$$

$$v_m = \frac{1}{\left\{\frac{\mu\varepsilon}{2}(1+1)\right\}^{1/2}} = \frac{1}{\sqrt{\mu\varepsilon}} \quad \text{(0.4 pts)}$$

**2.** The maximum depth of detection (skin depth, $\delta$) of an object in the ground is inversely proportional to the attenuation constant:

**(0.5 pts)** **(0.3 pts)** **(0.2 pts)**

$$\delta = \frac{1}{\alpha} = \frac{1}{\omega\left\{\frac{\mu\varepsilon}{2}\left[\left(1+\frac{\sigma^2}{\varepsilon^2\omega^2}\right)^{1/2}-1\right]\right\}^{1/2}} = \frac{1}{\omega\left\{\frac{\mu\varepsilon}{2}\left[\left(1+\frac{1}{2}\frac{\sigma^2}{\varepsilon^2\omega^2}\right)-1\right]\right\}^{1/2}} = \frac{1}{\omega\left\{\frac{\mu\varepsilon}{2}.\frac{1}{2}\frac{\sigma^2}{\varepsilon^2\omega^2}\right\}^{1/2}}$$

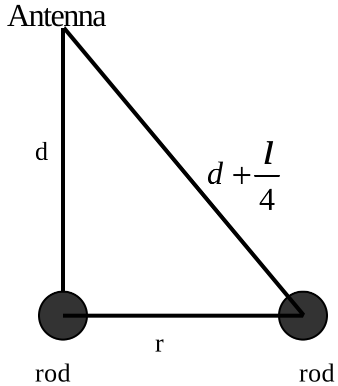
$$\delta = \left(\frac{2}{\sigma}\right)\left(\frac{\varepsilon}{\mu}\right)^{1/2}.$$

Numerically $\delta = \frac{\left(5.31\sqrt{\varepsilon_r}\right)}{\sigma}$ m, where $\sigma$ is in mS/m. **(0.5 pts)**

For a medium with conductivity of 1.0 mS/m and relative permittivity of 9, the skin depth

$$\delta = \frac{\left(5.31\sqrt{9}\right)}{1.0} = 15.93 \text{ m}$$ **(0.3 pts) + (0.2 pts)**

**3.** Lateral resolution:




$$r^2+d^2=(d+\frac{\lambda}{4})^2$$

$$r=\left(\frac{\lambda d}{2}+\frac{\lambda^2}{16}\right)^{1/2}$$

**(1.0 pts)**

r =0.5 m, d =4 m: $\frac{1}{2}=\left(\frac{4\lambda}{2}+\frac{\lambda^2}{16}\right)^{1/2}$, $\lambda^2+32\lambda-4=0$ **(0.5 pts)**

The wavelength is λ=0.125 m. **(0.3 pts) + (0.2 pts)**

The propagation speed of the signal in medium is

$$v_m=\frac{1}{\sqrt{\mu\varepsilon}}=\frac{1}{\sqrt{\mu_o\mu_r\varepsilon_o\varepsilon_r}}=\frac{1}{\sqrt{\mu_o\varepsilon_o}}\frac{1}{\sqrt{\mu_r\varepsilon_r}}$$

$$v_m=\frac{c}{\sqrt{\mu_r\varepsilon_r}}=\frac{0.3}{\sqrt{\varepsilon_r}}\text{ m/ns}\quad,\ \text{where }\ c=\frac{1}{\sqrt{\mu_o\varepsilon_o}}\text{ and }\ \mu_{\mathrm{r}}=1$$

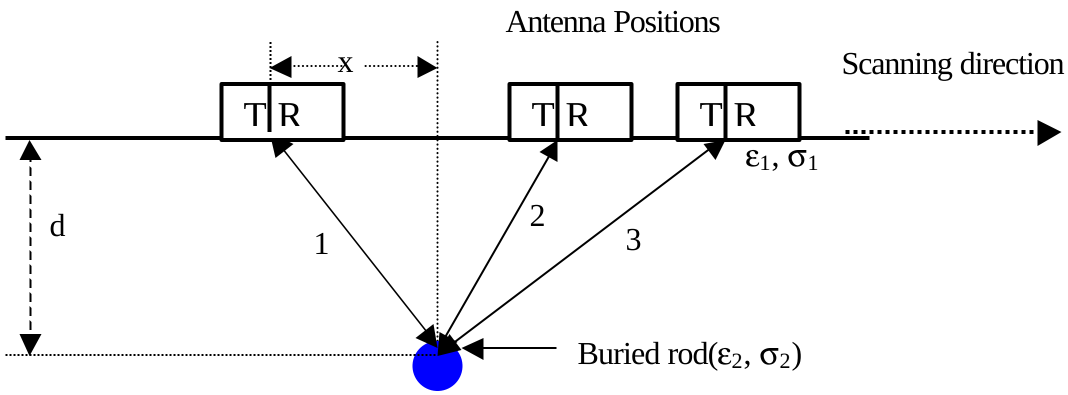
$$v_m=0.1\text{ m/ns}=10^8\text{ m/s}$$ **(0.5 pts)**

The minimum frequency need to distinguish the two rods as two separate objects is

$$f_{\min}=\frac{v}{\lambda}$$ **(0.5 pts)**

$$f_{\min}=\frac{\frac{0.3}{\sqrt{9}}}{0.125}x10^9\ \text{ Hz }=800\text{ MHz}$$ **(0.3 pts) + (0.20 pts)**

**4.** Path of EM waves for some positions on the ground surface

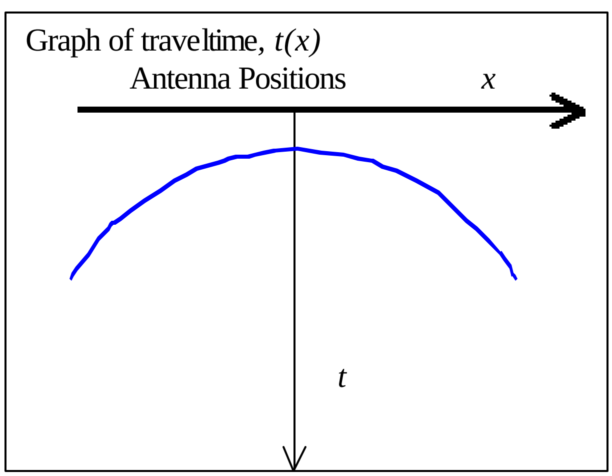



The traveltime as function of $x$ is

$$\left(\frac{t\ v}{2}\right)^2 = d^2 + x^2 ,$$ **(1.0 pts)**

$$t(x) = \sqrt{\frac{4d^2 + 4x^2}{v}}$$ **(1.0 pts)**

$$t(x) = \frac{2\sqrt{e_{1r}}}{0.3}\sqrt{d^2 + x^2}$$




For $x$ =0 **(1.0 pts)**

100 = 2×(3/0.3) $d$

$d$ = 5 m **(0.5 pts)**