Theoretical Competition

*Monday, July 2nd, 2001*

Please read this first:

1. The time available is 5 hours for the theoretical competition.
2. Use only the pen provided.
3. Use only the front side of the paper.
4. Begin each part of the problem on a separate sheet.
5. For each question, in addition to the ***blank sheets*** where you may write, there is an **answer form** where you *must* summarize the results you have obtained. Numerical results should be written with as many digits as are appropriate to the given data.
6. Write on the blank sheets of paper whatever you consider is required for the solution of the question. Please use *as little text as possible*; express yourself primarily in equations, numbers, figures, and plots.
7. Fill in the boxes at the top of each sheet of paper used by writing your ***Country No*** and ***Country Code,*** your student number (***Student No***), the number of the question (***Question No)***, the progressive number of each sheet ***(Page No),*** and the total number of blank sheets used for each question ***(Total No of pages).*** Write the question number and the section letter of the part you are answering at the top of each sheet. If you use some blank sheets of paper for notes that you do not wish to be marked, put a large X across the entire sheet and do not include it in your numbering.
8. At the end of the exam, arrange all sheets for each problem *in the following order;*
   - answer form
   - used sheets in order
   - the sheets you do not wish to be marked
   - unused sheets and the printed question

   Place the papers inside the envelope and leave everything on your desk. You are not allowed to take *any* sheets of paper out of the room.

Question 1

1a) KLYSTRON

Klystrons are devices used for amplifying very high-frequency signals. A klystron basically consists of two identical pairs of parallel plates (cavities) separated by a distance b, as shown in the figure.
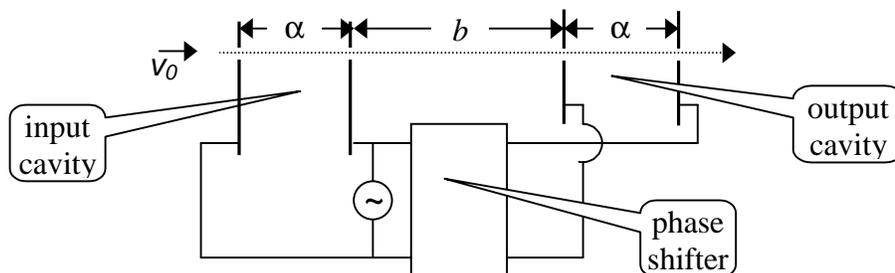
An electron beam with an initial speed v_0 traverses the entire system, passing through small holes in the plates. The high-frequency voltage to be amplified is applied to both pairs of plates with a certain phase difference (where period T corresponds to 2π phase) between them, producing horizontal, alternating electric fields in the cavities. The electrons entering the input cavity when the electric field is to the right are retarded and vice versa, so that the emerging electrons form bunches at a certain distance. If the output cavity is placed at the bunching point, the electric field in this cavity will absorb power from the beam provided that its phase is appropriately chosen. Let the voltage signal be a square wave with period $T=1.0\times10^{-9}$ s, changing between $V=\pm0.5$ volts. The initial velocity of the electrons is $v_0=2.0\times10^6$ m/s and the charge to mass ratio is $e/m=1.76\times10^{11}$ C/kg. The distance α is so small that the transit time in the cavities can be neglected. Keeping 4 significant figures, calculate;

a) the distance b, where the electrons bunch. Copy your result onto the **answer form.** *[1.5 pts]*

b) the necessary phase difference to be provided by the phase shifter. Copy your result onto the **answer form.** *[1.0 pts]*

1b) INTERMOLECULAR DISTANCE

Let d_L and d_V represent the average distances between molecules of water in the liquid phase and in the vapor phase, respectively. Assume that both phases are at 100 °C and atmospheric pressure, and the vapor behaves like an ideal gas. Using the following data, calculate the ratio d_V/d_L and copy your result onto the **answer form.** *[2.5 pts]*

Density of water in liquid phase: $\rho_L=1.0\times10^3$ kg/m^3,

Molar mass of water: $M=1.8\times10^{-2}$ kg/mol
Atmospheric pressure: $P_a=1.0\times10^5$ N/m^2
Gas constant: $R=8.3$ J/mol· K
Avagadro's number: $N_A=6.0\times10^{23}$ /mol

1c) SIMPLE SAWTOOTH SIGNAL GENERATOR

A sawtooth voltage waveform V_0 can be obtained across the capacitor C in Fig. 1. R is a variable resistor, V_i is an ideal battery, and SG is a spark gap consisting of two electrodes with an adjustable distance between them. When the voltage across the electrodes exceeds the firing voltage V_f, the air between the electrodes breaks down, hence the gap becomes a short circuit and remains so until the voltage across the gap becomes very small.
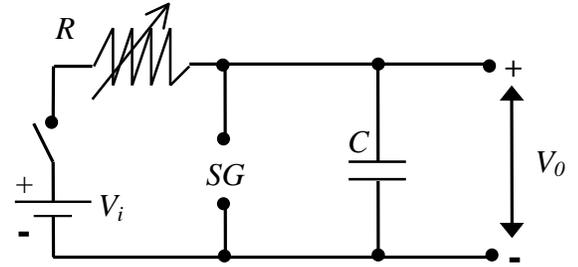
Figure 1

a) Draw the voltage waveform V_0 versus time t, after the switch is closed. *[0.5 pts]*

b) What condition must be satisfied in order to have an almost linearly varying sawtooth voltage waveform V_0? Copy your result onto the **answer form**. *[0.2 pts]*

c) Provided that this condition is satisfied, derive a simplified expression for the period T of the waveform. Copy your result onto the **answer form.** *[0.4 pts]*

d) What should you vary(R and/or SG) to change the period only? Copy your result onto the **answer form.** *[0.2 pts]*

e) What should you vary (R and/or SG) to change the amplitude only? Copy your result onto the **answer form.** *[0.2 pts]*

f) You are given an additional, adjustable DC voltage supply. Design and draw a new circuit indicating the terminals where you would obtain the voltage waveform V_0' described in Fig. 2. *[1.0 pts]*
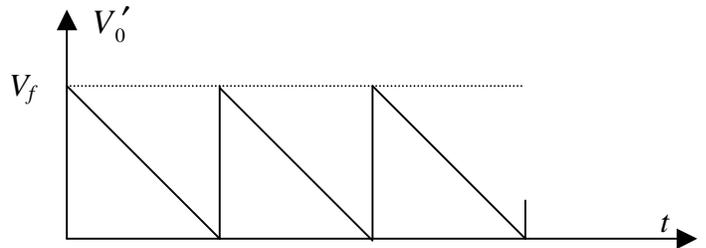
Figure 2

1d) ATOMIC BEAM

An atomic beam is prepared by heating a collection of atoms to a temperature T and allowing them to emerge horizontally through a small hole (of atomic dimensions) of diameter D in one side of the oven. Estimate the diameter of the beam after it has traveled a horizontal length L along its path. The mass of an atom is M. Copy your result onto the **answer form.** *[2.5 pts]*

Diameter $= D$

L

Oven at temperature T

Atoms of mass M
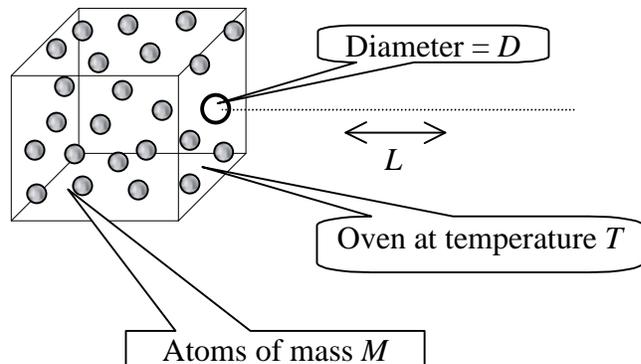
Question 2

BINARY STAR SYSTEM

a) It is well known that most stars form binary systems. One type of binary system consists of an ordinary star with mass m_0 and radius R, and a more massive, compact neutron star with mass M, rotating around each other. In all the following ignore the motion of the earth. Observations of such a binary system reveal the following information:

- The maximum angular displacement of the ordinary star is $\Delta\theta$, whereas that of the neutron star is $\Delta\phi$ (see Fig. 1).
- The time it takes for these maximum displacements is τ.
- The radiation characteristics of the ordinary star indicate that its surface temperature is T and the radiated energy incident on a unit area on earth's surface per unit time is P.
- The calcium line in this radiation differs from its normal wavelength λ_0 by an amount $\Delta\lambda$, due only to the gravitational field of the ordinary star. (For this calculation the photon can be considered to have an effective mass of $h/c\lambda$.)
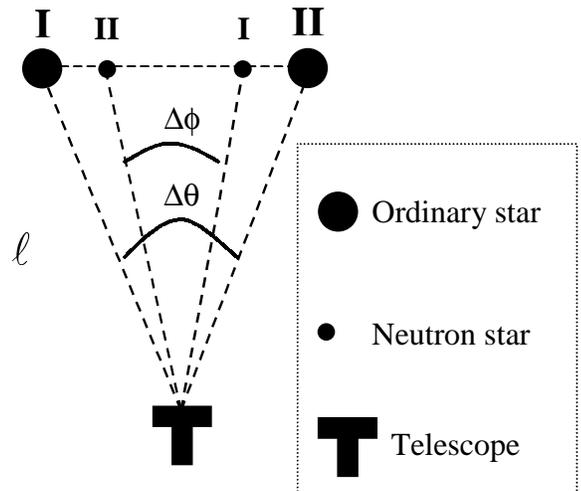

Fig. 1

Find an expression for the distance ℓ from earth to this system, only in terms of the observed quantities and universal constants. Copy your result onto the **answer form**. *[7 pts]*

b) Assume that $M \gg m_0$, so that the ordinary star is basically rotating around the neutron star in a circular orbit of radius r_0. Assume that the ordinary star starts emitting gas toward the neutron star with a speed v_0, relative to the ordinary star (see Fig. 2). Assuming that the neutron star is the dominant gravitational force in this problem and neglecting the orbital changes of the ordinary star find the distance of closest approach r_f shown in Fig. 2. Copy your result onto the **answer form**. *[3pts]*
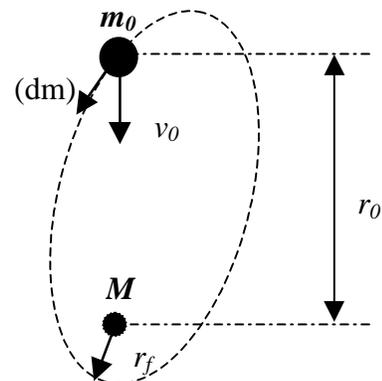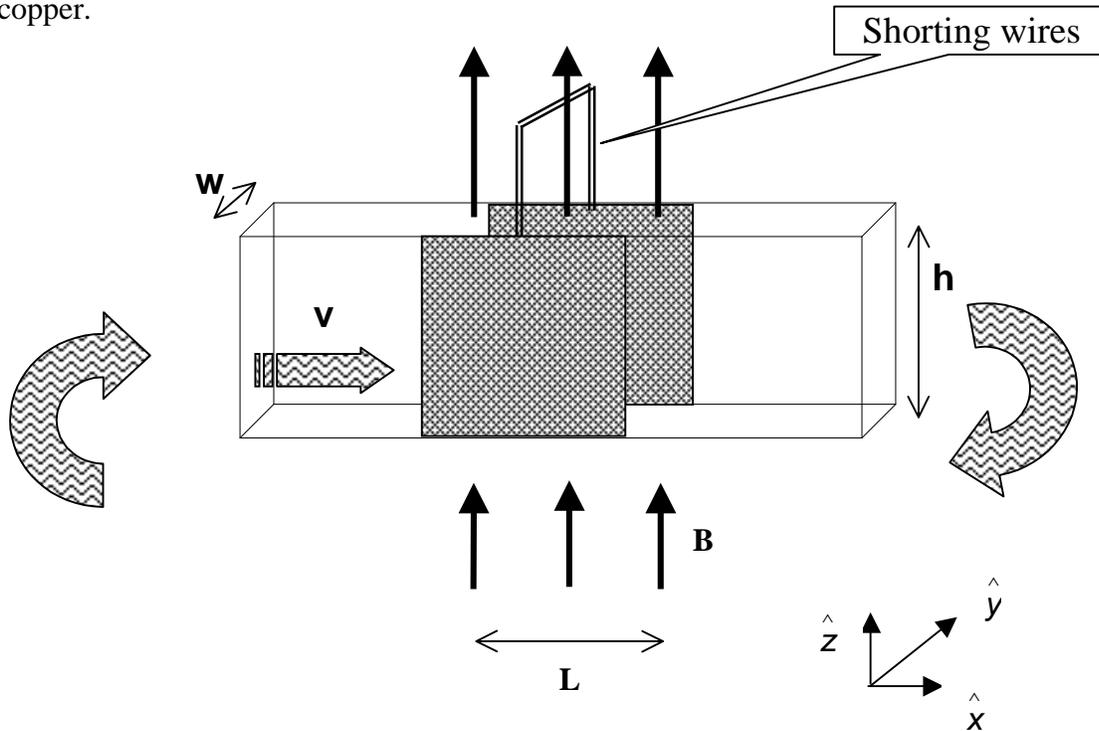

Fig. 2

Question 3

MAGNETOHYDRODYNAMIC (MHD) GENERATOR

A horizontal rectangular plastic pipe of width w and height h, which closes upon itself, is filled with mercury of resistivity ρ. An overpressure P is produced by a turbine which drives this fluid with a constant speed v_0. The two opposite vertical walls of a section of the pipe with length L are made of copper.

The motion of a real fluid is very complex. To simplify the situation we assume the following:
- Although the fluid is viscous, its speed is uniform over the entire cross section.
- The speed of the fluid is always proportional to the net external force acting upon it.
- The fluid is incompressible.

These walls are electrically shorted externally and a uniform, magnetic field **B** is applied vertically upward only in this section. The set up is illustrated in the figure above, with the unit vectors $\hat{x}$, $\hat{y}$, $\hat{z}$ to be used in the solution.

a) Find the force acting on the fluid due to the magnetic field (in terms of L, B, h, w, ρ and the new velocity v) [2.0 pts]

b) Derive an expression for the new speed v of the fluid (in terms of v_0, P, L, B and ρ) after the magnetic field is applied. [3.0 pts]

c) Derive an expression for the additional power that must be supplied by the turbine to increase the speed to its original value v_0. Copy your result onto the **answer form.** [2.0 pts]

d) Now the magnetic field is turned off and mercury is replaced by water flowing with speed v_0. An electromagnetic wave with a single frequency is sent along the section with length L in the direction of the flow. The refractive index of water is n, and $v_0 \ll c$. Derive an expression for the contribution of the fluid's motion to the phase difference between the waves entering and leaving section L. Copy your result onto the **answer form.** [3.0 pts]

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ANSWER FORM

1A

a)

$b =$

b)

Phase difference=

1B

$\dfrac{d_V}{d_L} =$

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1C

b)

c)

$T=$

d)

e)

1D

New diameter of the beam =

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ANSWER FORM

2a)

$\ell =$

2b)

$r_f =$

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ANSWER FORM

3a)

3b)

v =

3c)

Power =

3d)

Phase difference =