Problems of the 18th International Physics Olympiad
(Jena, 1987)

Gunnar Friege & Gunter Lind[1]

Leibniz-Institute for Science Education (IPN) at the University of Kiel, Germany

Abstract

The 18th International Physics Olympiad took place in 1987 in the German Democratic Republic (GDR). This article contains the competition problems, their solutions and also a (rough) grading scheme.

Introduction

The 18th international Physics olympics in 1987 was the second International Physics Olympiad hosted by the German Democratic Republic (GDR) . The organisation was lead by the ministry for education and the problems were formulated by a group of professors of different universities. However, the main part of the work was done by the physics department of the university of Jena. The company Carl-Zeiss and a special scientific school in Jena were involved also.

In the competition three theoretical and one experimental problem had to be solved. The theoretical part was quite difficult. Only the first of the three problems (“ascending moist air”) had a medium level of difficulty. The points given in the markings were equal distributed. Therefore, there were lots of good but also lots of unsatisfying solutions. The other two theoretical problems were rather difficult. About half of the pupils even did not find an adequate start in solving these problems. The third problem (“infinite LC-grid") revealed quite a few complete solutions. The high level of difficulty can probably be explained with the fact that many pupils nearly had no experience with the subject. Concerning the second problem (“electrons in a magnetic field”) only a few pupils worked on the last part 3 (see below).

The experimental problem (“refracting indices”) was much more easier than the theoretical problems. There were lots of different possibilities of solution and most of the pupils

[1] Contact: Leibniz-Institute for Science Education (IPN) at the University of Kiel
Olshausenstrasse 62, 24098 Kiel, Germany
ipho@ipn.uni-kiel.de

managed to come up with partial or complete solutions. Over the half of all teams got more points in the experimental part than in the theoretical part of the competition.

The problems and their solutions are based on the original German and English versions of the competition problems. Only minor changes have been made. Despite the fact that nowadays almost all printed figures are generated with the aid of special computer programmes, the original hand-made figures are published here.

Theoretical Problems

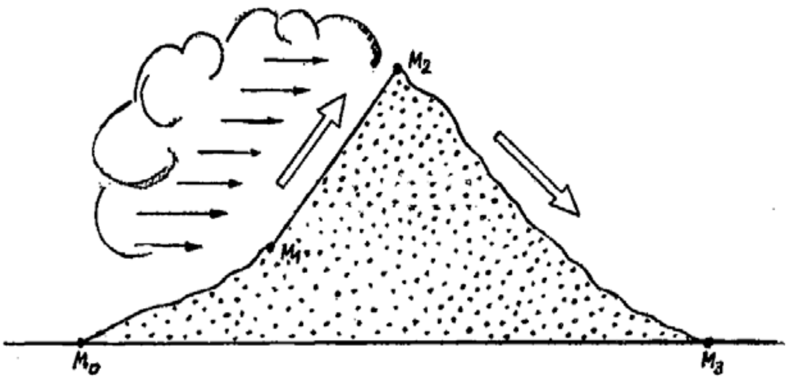
Problem 1: Ascending moist air

Moist air is streaming adiabatically across a mountain range as indicated in the figure.

Equal atmospheric pressures of 100 kPa are measured at meteorological stations M_0 and M_3 and a pressure of 70 kPa at station M_2. The temperature of the air at M_0 is 20° C.

As the air is ascending, cloud formation sets in at 84.5 kPa.

Consider a quantity of moist air ascending the mountain with a mass of 2000 kg over each square meter. This moist air reaches the mountain ridge (station M_2) after 1500 seconds. During that rise an amount of 2.45 g of water per kilogram of air is precipitated as rain.



1. Determine temperature T_1 at M_1 where the cloud ceiling forms.
2. What is the height h_1 (at M_1) above station M_0 of the cloud ceiling assuming a linear decrease of atmospheric density?
3. What temperature T_2 is measured at the ridge of the mountain range?
4. Determine the height of the water column (precipitation level) precipitated by the air stream in 3 hours, assuming a homogeneous rainfall between points M_1 and M_2.

5. What temperature T_3 is measured in the back of the mountain range at station M_3? Discuss the state of the atmosphere at station M_3 in comparison with that at station M_0.

Hints and Data

The atmosphere is to be dealt with as an ideal gas. Influences of the water vapour on the specific heat capacity and the atmospheric density are to be neglected; the same applies to the temperature dependence of the specific latent heat of vaporisation. The temperatures are to be determined to an accuracy of 1 K, the height of the cloud ceiling to an accuracy of 10 m and the precipitation level to an accuracy of 1 mm.

Specific heat capacity of the atmosphere in the pertaining temperature range:

$c_p = 1005\ \text{J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}$

Atmospheric density for p_0 and T_0 at station M_0: $\rho_0 = 1.189\ \text{kg} \cdot \text{m}^{-3}$

Specific latent heat of vaporisation of the water within the volume of the cloud:

$L_v = 2500\ \text{kJ} \cdot \text{kg}^{-1}$

$\frac{c_p}{c_v} = \chi = 1.4$ and $g = 9.81\ \text{m} \cdot \text{s}^{-2}$

Solution of problem 1:

1. Temperature T_1 where the cloud ceiling forms

$$T_1 = T_0 \cdot \left(\frac{p_1}{p_0}\right)^{1-\frac{1}{\chi}} = 279\,\text{K} \tag{1}$$

2. Height h_1 of the cloud ceiling:

$$p_0 - p_1 = \frac{\rho_0 + \rho_1}{2} \cdot g \cdot h_1, \text{ with } \rho_1 = \rho_0 \cdot \frac{p_1}{p_0} \cdot \frac{T_0}{T_1}.$$

$$h_1 = 1410\,\text{m} \tag{2}$$

3. Temperature T_2 at the ridge of the mountain.

   The temperature difference when the air is ascending from the cloud ceiling to the mountain ridge is caused by two processes:

   – adiabatic cooling to temperature T_x,

– heating by ΔT by condensation.

$$T_2 = T_x + \Delta T \quad (3)$$

$$T_x = T_1 \cdot \left(\frac{p_2}{p_1}\right)^{1-\frac{1}{\chi}} = 265\,\mathrm{K} \quad (4)$$

For each kg of air the heat produced by condensation is $L_v \cdot 2.45$ g = 6.125 kJ.

$$\Delta T = \frac{6.125}{c_p} \cdot \frac{\mathrm{kJ}}{\mathrm{kg}} = 6.1\,\mathrm{K} \quad (5)$$

$$T_2 = 271\ \mathrm{K} \quad (6)$$

4. Height of precipitated water column

$$h = 35\ \mathrm{mm} \quad (7)$$

5. Temperature T_3 behind the mountain

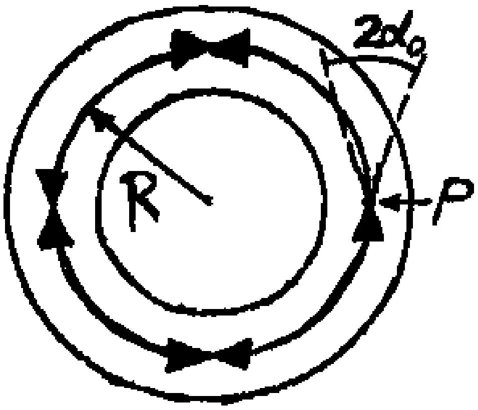
$$T_3 = T_2 \cdot \left(\frac{p_3}{p_2}\right)^{1-\frac{1}{\chi}} = 300\,\mathrm{K} \quad (8)$$

The air has become warmer and dryer. The temperature gain is caused by condensation of vapour.

Problem 2: Electrons in a magnetic field

A beam of electrons emitted by a point source P enters the magnetic field $\vec{B}$ of a toroidal coil (toroid) in the direction of the lines of force. The angle of the aperture of the beam $2 \cdot \alpha_0$ is assumed to be small ($2 \cdot \alpha_0 << 1$). The injection of the electrons occurs on the mean radius R of the toroid with acceleration voltage V_0.

Neglect any interaction between the electrons. The magnitude of $\vec{B}$, B, is assumed to be constant.



1. To guide the electron in the toroidal field a homogeneous magnetic deflection field $\vec{B}_1$ is required. Calculate $\vec{B}_1$ for an electron moving on a circular orbit of radius R in the torus.

2. Determine the value of $\vec{B}$ which gives four focussing points separated by $\pi/2$ as indicated in the diagram.

   Note: When considering the electron paths you may disregard the curvature of the magnetic field.

3. The electron beam cannot stay in the toroid without a deflection field $\vec{B}_1$, but will leave it with a systematic motion (drift) perpendicular to the plane of the toroid.

   a) Show that the radial deviation of the electrons from the injection radius is finite.

   b) Determine the direction of the drift velocity.

   Note: The angle of aperture of the electron beam can be neglected. Use the laws of conservation of energy and of angular momentum.

   Data:

   $\frac{e}{m} = 1.76 \cdot 10^{11}\,\mathrm{C \cdot kg^{-1}}$; $V_0 = 3$ kV; R = 50 mm

Solution of problem 2:

1. Determination of B:

   The vector of the velocity of any electron is divided into components parallel with and perpendicular to the magnetic field $\vec{B}$:

$$\vec{v} = \vec{v}_{\parallel} + \vec{v}_{\perp} \tag{1}$$

   The Lorentz force $\vec{F} = -e \cdot (\vec{v} \times \vec{B})$ influences only the perpendicular component, it acts as a radial force:

$$m \cdot \frac{v_{\perp}^2}{r} = e \cdot v_{\perp} \cdot B \tag{2}$$

   Hence the radius of the circular path that has been travelled is

$$r = \frac{m}{e} \cdot \frac{v_{\perp}}{B} \tag{3}$$

and the period of rotation which is independent of $v_\perp$ is

$$T = \frac{2 \cdot \pi \cdot r}{v_\perp} = \frac{2 \cdot \pi \cdot m}{B \cdot e} \tag{4}$$

The parallel component of the velocity does not vary. Because of $\alpha_0 << 1$ it is approximately equal for all electrons:

$$v_{\parallel 0} = v_0 \cdot \cos\alpha_0 \approx v_0 \tag{5}$$

Hence the distance b between the focusing points, using eq. (5), is

$$b = v_{\parallel 0} \cdot T \approx v_0 \cdot T \tag{6}$$

From the law of conservation of energy follows the relation between the acceleration voltage V_0 and the velocity v_0:

$$\frac{m}{2} \cdot {v_0}^2 = e \cdot V_0 \tag{7}$$

Using eq. (7) and eq. (4) one obtains from eq. (6)

$$b = \frac{2 \cdot \pi}{B} \cdot \sqrt{2 \cdot \frac{m}{e} \cdot V_0} \tag{8}$$

and because of $b = \frac{2 \cdot \pi \cdot R}{4}$ one obtains

$$B = \frac{4}{R} \cdot \sqrt{2 \cdot \frac{m}{e} \cdot V_0} = 1.48 \cdot 10^{-2} \frac{Vs}{m^2} \tag{9}$$

2. Determination of B_1:

Analogous to eq. (2)

$$m \cdot \frac{v_0^2}{R} = e \cdot v_0 \cdot B_1 \tag{10}$$

must hold.

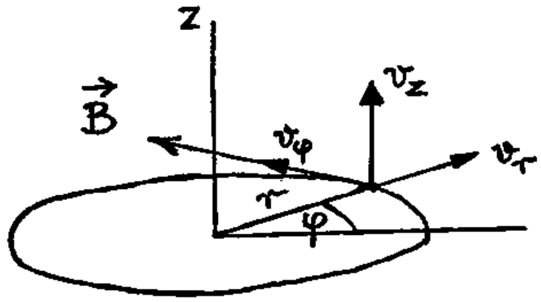
From eq. (7) follows

$$B_1 = \frac{1}{R} \cdot \sqrt{2 \cdot \frac{m}{e} \cdot V_0} = 0.37 \cdot 10^{-2} \frac{Vs}{m^2} \tag{11}$$

3. Finiteness of r_1 and direction of the drift velocity

In the magnetic field the lines of force are circles with their centres on the symmetry axis (z-axis) of the toroid.

In accordance with the symmetry of the problem, polar coordinates r and φ are introduced into the plane perpendicular to the z-axis (see figure below) and the occurring vector quantities (velocity, magnetic field $\vec{B}$, Lorentz force) are divided into the corresponding components.



Since the angle of aperture of the beam can be neglected examine a single electron injected tangentially into the toroid with velocity v_0 on radius R.

In a static magnetic field the kinetic energy is conserved, thus

$$E = \frac{m}{2}\left(v_r^2 + v_\varphi^2 + v_z^2\right) = \frac{m}{2} v_0^2 \qquad (12)$$

The radial points of inversion of the electron are defined by the condition

$$v_r = 0$$

Using eq. (12) one obtains

$$v_0^2 = v_\varphi^2 + v_z^2 \qquad (13)$$

Such an inversion point is obviously given by

$$r = R \cdot \left(v_\varphi = v_0,\ v_r = 0,\ v_z = 0\right).$$

To find further inversion points and thus the maximum radial deviation of the electron the components of velocity v_φ and v_z in eq. (13) have to be expressed by the radius.

v_φ will be determined by the law of conservation of angular momentum. The Lorentz force obviously has no component in the φ - direction (parallel to the magnetic field). Therefore it cannot produce a torque around the z-axis. From this follows that the z-component of the angular momentum is a constant, i.e. $L_z = m \cdot v_\varphi \cdot r = m \cdot v_0 \cdot R$ and

therefore $v_\varphi = v_0 \cdot \frac{R}{r}$ (14)

v_z will be determined from the equation of motion in the z-direction. The z-component of the Lorentz force is $F_z = - e \cdot B \cdot v_r$. Thus the acceleration in the z-direction is

$$a_z = -\frac{e}{m} \cdot B \cdot v_r \,. \qquad (15).$$

That means, since B is assumed to be constant, a change of v_z is related to a change of r as follows:

$$\Delta v_z = -\frac{e}{m} \cdot B \cdot \Delta r$$

Because of $\Delta r = r - R$ and $\Delta v_z = v_z$ one finds

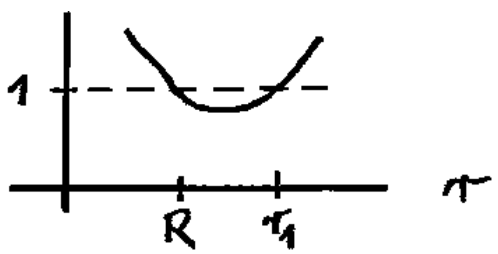
$$v_z = -\frac{e}{m} \cdot B \cdot (r - R) \qquad (16)$$

Using eq. (14) and eq. (15) one obtains for eq. (13)

$$1 = \left(\frac{R}{r}\right)^2 + A^2 \cdot \left(\frac{r}{R} - 1\right)^2 \qquad (17)$$

where $A = \frac{e}{m} \cdot B \cdot \frac{R}{v_0}$

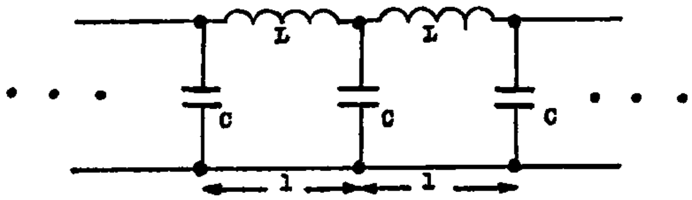
Discussion of the curve of the right side of eq. (17) gives the qualitative result shown in the following diagram:



Hence r_1 is finite. Since $R \le r \le r_1$ eq. (16) yields $v_z < 0$. Hence the drift is in the direction of the negative z-axis.

Problem 3: Infinite LC-grid

When sine waves propagate in an infinite LC-grid (see the figure below) the phase of the ac-voltage across two successive capacitors differs by Φ.



a) Determine how Φ depends on ω, L and C (ω is the angular frequency of the sine wave).

b) Determine the velocity of propagation of the waves if the length of each unit is ℓ .

c) State under what conditions the propagation velocity of the waves is almost independent of ω. Determine the velocity in this case.

d) Suggest a simple mechanical model which is an analogue to the above circuit and derive equations which establish the validity of your model.

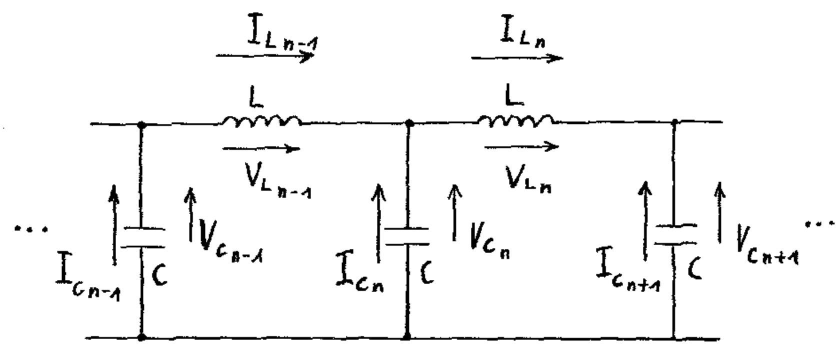
Formulae:

$$\cos\alpha - \cos\beta = -2 \cdot \sin\left(\frac{\alpha+\beta}{2}\right) \cdot \sin\left(\frac{\alpha-\beta}{2}\right)$$

$$\sin\alpha - \sin\beta = 2 \cdot \cos\left(\frac{\alpha+\beta}{2}\right) \cdot \sin\left(\frac{\alpha-\beta}{2}\right)$$

Solution of problem 3:

a)



Current law: $I_{L_{n-1}} + I_{C_n} - I_{L_n} = 0$ (1)

Voltage law: $V_{C_{n-1}} + V_{L_{n-1}} - V_{C_n} = 0$ (2)

Capacitive voltage drop: $V_{C_{n-1}} = \frac{1}{\omega \cdot C} \cdot \tilde{I}_{C_{n-1}}$ (3)

Note: In eq. (3) $\tilde{I}_{C_{n-1}}$ is used instead of $I_{C_{n-1}}$ because the current leads the voltage by 90°.

Inductive voltage drop: $V_{L_{n-1}} = \omega \cdot L \cdot \tilde{I}_{L_{n-1}}$ (4)

Note: In eq. (4) $\tilde{I}_{L_{n-1}}$ is used instead of $I_{L_{n-1}}$ because the current lags behind the voltage by 90°.

The voltage V_{C_n} is given by: $V_{C_n} = V_0 \cdot \sin(\omega \cdot t + n \cdot \varphi)$ (5)

Formula (5) follows from the problem.

From eq. (3) and eq. (5): $I_{C_n} = \omega \cdot C \cdot V_0 \cdot \cos(\omega \cdot t + n \cdot \varphi)$ (6)

From eq. (4) and eq. (2) and with eq. (5)

$$I_{L_{n-1}} = \frac{V_0}{\omega \cdot L} \cdot \left[2 \cdot \sin\left(\omega \cdot t + \left(n - \frac{1}{2} \right) \cdot \varphi \right) \cdot \sin\frac{\varphi}{2} \right] \qquad (7)$$

$$I_{L_n} = \frac{V_0}{\omega \cdot L} \cdot \left[2 \cdot \sin\left(\omega \cdot t + \left(n + \frac{1}{2} \right) \cdot \varphi \right) \cdot \sin\frac{\varphi}{2} \right] \qquad (8)$$

Eqs. (6), (7) and (8) must satisfy the current law. This gives the dependence of φ on ω, L and C.

$$0 = V_0 \cdot \omega \cdot C \cdot \cos(\omega \cdot t + n \cdot \varphi) + 2 \cdot \frac{V_0}{\omega \cdot L} \cdot \sin\frac{\varphi}{2} \cdot \left[2 \cdot \cos(\omega \cdot t + n \cdot \varphi) \cdot \sin\left(-\frac{\varphi}{2} \right) \right]$$

This condition must be true for any instant of time. Therefore it is possible to divide by $V_0 \cdot \cos(\omega \cdot t + n \cdot \varphi)$.

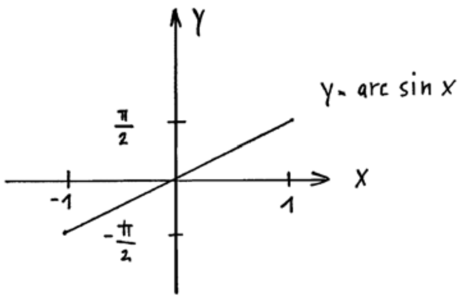
Hence $\omega^2 \cdot L \cdot C = 4 \cdot \sin^2\left(\frac{\varphi}{2}\right)$. The result is

$$\varphi = 2 \cdot \arcsin\left(\frac{\omega \cdot \sqrt{L \cdot C}}{2} \right) \quad \text{with} \quad 0 \le \omega \le \frac{2}{\sqrt{L \cdot C}} \qquad (9).$$

b) The distance ℓ is covered in the time Δt thus the propagation velocity is

$$v = \frac{\ell}{\Delta t} = \frac{\omega \cdot \ell}{\varphi} \quad \text{or} \quad v = \frac{\omega \cdot \ell}{2 \cdot \arcsin\left(\frac{\omega \cdot \sqrt{L \cdot C}}{2} \right)} \qquad (10)$$

c)



Slightly dependent means arc sin $\left(\frac{\omega \cdot \sqrt{L \cdot C}}{2}\right) \sim \omega$, since v is constant in that case.

This is true only for small values of ω. That means $\frac{\omega \cdot \sqrt{L \cdot C}}{2} \ll 1$ and therefore

$$v_0 = \frac{\ell}{\sqrt{L \cdot C}} \tag{11}$$

d) The energy is conserved since only inductances and capacitances are involved. Using the terms of a) one obtains the capacitive energy

$$W_C = \sum_n \frac{1}{2} \cdot C \cdot V_{C_n}{}^2 \tag{12}$$

and the inductive energy

$$W_L = \sum_n \frac{1}{2} \cdot L \cdot I_{L_n}{}^2 \tag{13}$$

From this follows the standard form of the law of conservation of energy

$$W_C = \sum_n \frac{1}{2}\left(C \cdot V_{C_n}{}^2 + L \cdot I_{L_n}{}^2\right) \tag{14}$$

The relation to mechanics is not recognizable in this way since two different physical quantities (V_{C_n} and I_{L_n}) are involved and there is nothing that corresponds to the relation between the locus x and the velocity v = $\dot{x}$.

To produce an analogy to mechanics the energy has to be described in terms of the charge Q, the current I = $\dot{Q}$ and the constants L and C. For this purpose the voltage V_{C_n} has to be expressed in terms of the charges Q_{L_n} passing through the coil.

One obtains:

$$W = \sum_n \left[\underbrace{\frac{L}{2} \cdot \dot{Q}_{L_n}^{\ 2}}_{A} + \underbrace{\frac{1}{2 \cdot C} \left(Q_{L_n} - Q_{L_{n-1}} \right)^2}_{B} \right] \tag{15}$$

Mechanical analogue:

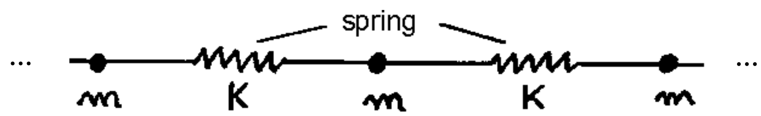
A (kinetic part): $\dot{Q}_{L_n} \longrightarrow v_n$; $L \longrightarrow m$

B (potential part): $Q_{L_n} \longrightarrow x_n$

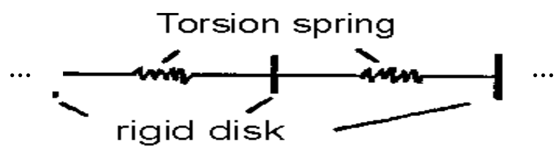
x_n: displacement and v_n: velocity.

However, Q_{L_n} could equally be another quantity (e.g. an angle). L could be e.g. a moment of inertia.

From the structure of the problems follows: Interaction only with the nearest neighbour (the force rises linearly with the distance). A possible model could be:



Another model is:



Experimental Problems

Problem 4: Refractive indices

Find the refractive indices of a prism, n_p, and a liquid, n_l. Ignore dispersion.

a) Determine the refractive index n_p of a single prism by two different experimental methods.
Illustrate your solution with accurate diagrams and deduce the relations necessary to calculate the refractive index. (One prism only should be used).

b) Use two identical prisms to determine the refractive index n_L of a liquid with $n_L < n_p$. Illustrate your solution with accurate diagrams and deduce the relations necessary to calculate the refractive index.

Apparatus:

Two identical prisms with angles of 30°, 60° and 90°; a set square, a glass dish, a round table, a liquid, sheets of graph paper, other sheets of paper and a pencil.

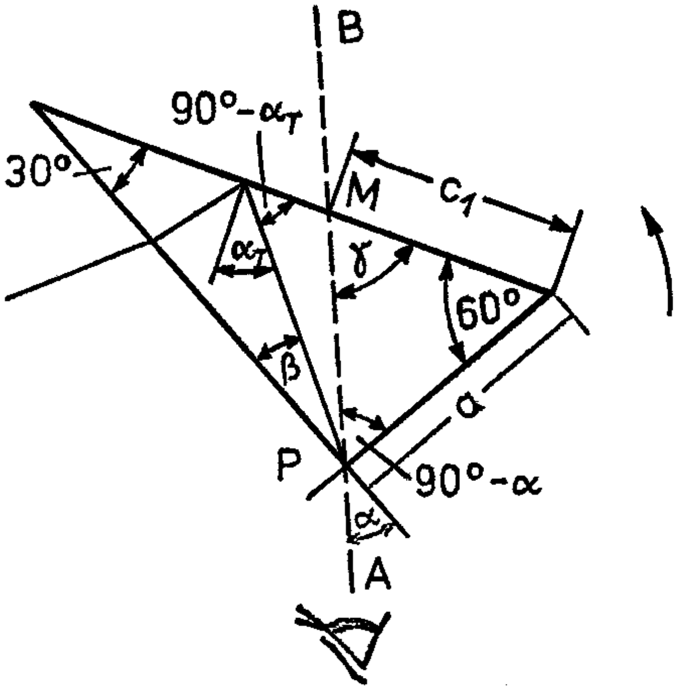
Formulae: $\sin(\alpha \pm \beta) = \sin\alpha \cdot \cos\beta \pm \cos\alpha \cdot \sin\beta$

Additional remarks: You may mark the opaque sides of the prisms with a pencil. The use of the lamp is optional.

Solution of problem 4:

a) Calculation of the refractive index of the prism

First method:



Draw a straight line A – B on a sheet of paper and let this be your line of sight. Place the prism with its rectangular edge facing you onto the line (at point P on the line). Now turn the prism in the direction of the arrow until the dark edge of total reflection which can be seen in the short face of the prism coincides with the 90° edge of the prism. Mark a point M and measure the length c_1. Measure also the length of the short face of the prism.

The following equations apply:

$$\sin\alpha_T = \frac{1}{n_p} \tag{1}$$

$$\frac{\sin\alpha}{\sin\beta} = n_p \tag{2}$$

$$\beta = 60° - \alpha_T \tag{3}$$

$$\gamma = 30° + \alpha \tag{4}$$

$$\frac{\sin\gamma}{\sin(90°-\alpha)} = \frac{a}{c_1} \tag{5}$$

From eq. (5) follows with eq. (4) and the given formulae:

$$\frac{a}{c_1}\cdot\cos\alpha = \sin(30°+\alpha) = \frac{1}{2}\cdot\cos\alpha + \frac{1}{2}\cdot\sqrt{3}\cdot\sin\alpha$$

$$\sin\alpha = \frac{2a - c_1}{2\cdot\sqrt{a^2 - a\cdot c_1 + c_1^{\,2}}} \tag{6}$$

From eqs. (2), (3) and (1) follows:

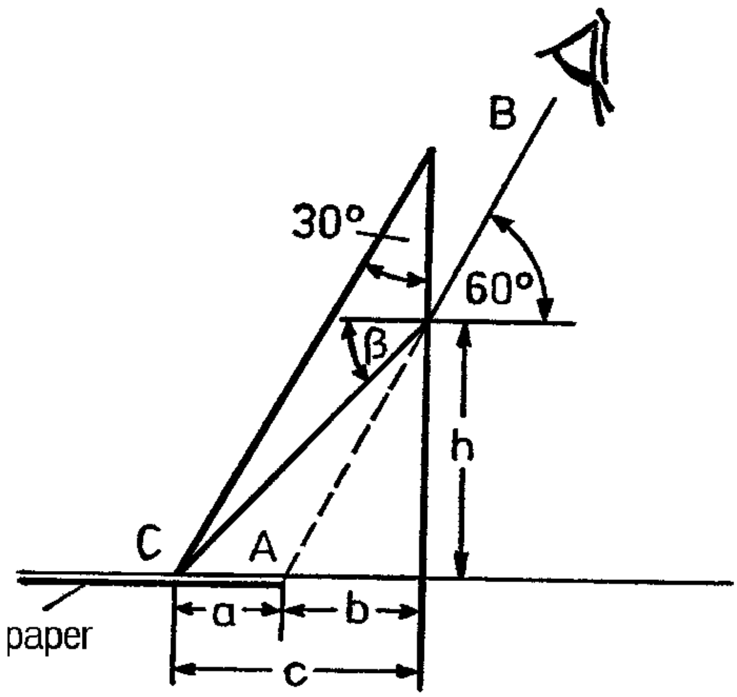
$$\sin\alpha = n_p\cdot\sin(60°-\alpha_T) = \frac{n_p}{2}\cdot\left(\sqrt{3}\cdot\cos\alpha_T - \sin\alpha_T\right)$$

$$n_p = +\left\{\frac{1}{3}\cdot(2\cdot\sin\alpha+1)^2+1\right\}^{1/2} \tag{7}$$

When measuring c_1 and a one notices that within the error limits of ± 1 mm a equals c_1.

Hence: $\sin\alpha = \frac{1}{2}$ and $n_p = 1.53$. (8)

Second method:



Place edge C of the prism on edge A of a sheet of paper and look along the prism hypotenuse at edge A so that your direction of sight B-A and the table surface form an angle of 60°. Then shift the prism over the edge of the paper into the position shown, such that prism edge C can be seen inside the prism collinear with edge A of the paper outside the prism. The direction of sight must not be changed while the prism is being displaced.

The following equations apply:

$$\left.\begin{array}{l}\tan\beta = \frac{h}{c}\\ \tan 60° = \sqrt{3} = \frac{h}{b}\end{array}\right\} => h = b\cdot\sqrt{3} = \frac{c\cdot\sin\beta}{\sqrt{1-\sin^2\beta}} \tag{9}$$

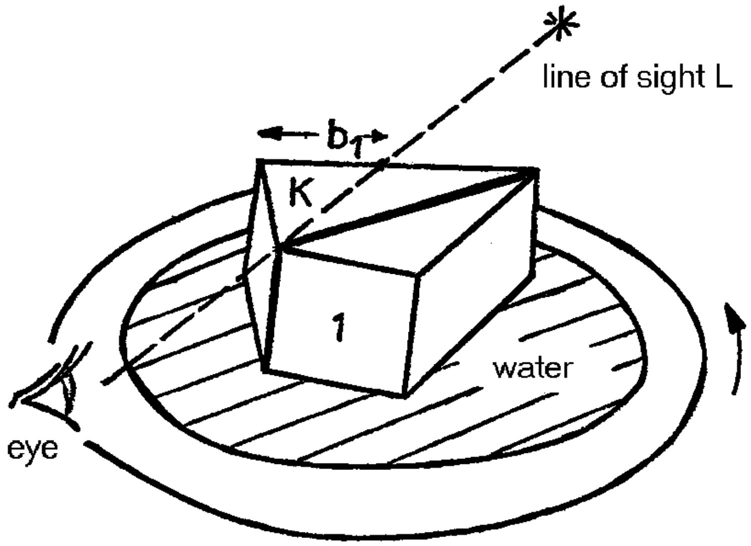
$$\sin\beta = \sin 60°\cdot\frac{1}{n_p} = \frac{\sqrt{3}}{2\cdot n_p} \tag{10}$$

$$n_p = \frac{1}{2}\cdot\sqrt{\left(\frac{c}{b}\right)^2 + 3} \tag{11}$$

With the measured values c = 29 mm and b = 11.5 mm, it follows

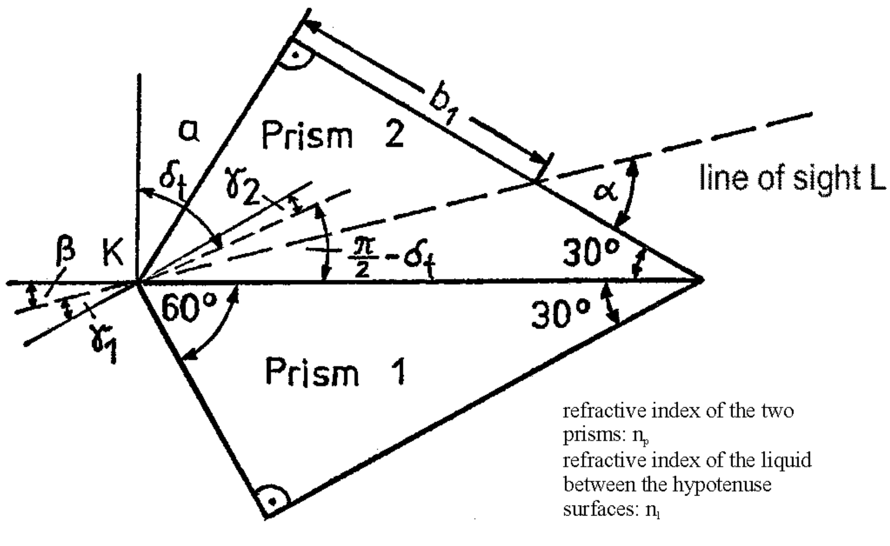
$$n_p = 1.53. \tag{12}$$

b) Determination of the refractive index of the liquid by means of two prisms



Place the two prisms into a glass dish filled with water as shown in the figure above. Some water will rise between the hypotenuse surfaces. By pressing and moving the prisms slightly against each other the water can be made to cover the whole surface. Look over the 60° edges of the prisms along a line of sight L (e.g. in the direction of a fixed point on an illuminated wall). Turn the glass dish together with the two prisms in such a way that the dark shadow of total reflection which can be seen in the short face of prism 1 coincides with the 60° edge of that prism (position shown in the figure below).

While turning the arrangement take care to keep the 60° edge (point K) on the line of sight L. In that position measure the length b_1 with a ruler (marking, reading). The figure below illustrates the position described.



If the refractive index of the prism is known (see part a) the refractive index of the liquid may be calculated as follows:

$$\sin\alpha = \frac{a}{\sqrt{a^2 + b_1^{\,2}}} \qquad (13)$$

$$\beta = \alpha - 30°;\ \ \gamma_1 = 30° - \beta = 60° - \alpha \qquad (14, 15)$$

$$\frac{\sin\gamma_1}{\sin\gamma_2} = n_p \quad \text{refraction at the short face of prism 1.} \qquad (16)$$

The angle of total reflection δ_t at the hypotenuse surface of prism 1 in the position described is:

$$\frac{\pi}{2} - \delta_t = 30° - \gamma_2 \qquad (17)$$

$$\delta_t = 60° + \arcsin\left(\frac{\sin\gamma_1}{n_p}\right) \qquad (18)$$

From this we can easily obtain n_1:

$$n_1 = n_p \cdot \sin\delta_t = n_p \cdot \sin\left\{60° + \arcsin\frac{\sin\gamma_1}{n_p}\right\} \qquad (19)$$

Numerical example for water as liquid:

b_1 = 1.9 cm; α = 55.84°; γ_1 = 4.16°; δ_t = 62.77°; a = 2.8 cm; with n_p = 1.5 follows

n_1= 1.33. (20)

Grading Scheme

Theoretical problems

Problem 1: Ascending moist art	
part 1	2
part 2	2
part 3	2
part 4	2
part 5	2
	10

Problem 2: Electron in a magnetic field	
part 1	3
part 2	1
part 3	6
	10

Problem 3: Infinite LC-grid	
part a	4
part b	1
part c	1
part d	4
	10

Problem 4: Refractive indices	
part a, first method	5
part a, second method	5
part b	10
	20