

Problems and solutions of the 16th IPhO* Portorož, Slovenia, (Former Yugoslavia), 1985

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1 Problems

1.1 Theoretical competition

Problem 1

A young radio amateur maintains a radio link with two girls living in two towns. He positions an aerial array such that when the girl living in town A receives a maximum signal, the girl living in town B receives no signal and vice versa. The array is built from two vertical rod aerials transmitting with equal intensities uniformly in all directions in the horizontal plane.

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- a) Find the parameters of the array, i. e. the distance between the rods, its orientation and the phase shift between the electrical signals supplied to the rods, such that the distance between the rods is minimum.
- b) Find the numerical solution if the boy has a radio station transmitting at 27 MHz and builds up the aerial array at Portorož. Using the map he has found that the angles between the north and the direction of A (Koper) and of B (small town of Buje in Istria) are 72° and 157° , respectively.

Problem 2

In a long bar having the shape of a rectangular parallelepiped with sides a , b , and c ($a \gg b \gg c$), made from the semiconductor InSb flows a current I parallel to the edge a . The bar is in an external magnetic field B which is parallel to the edge c . The magnetic field produced by the current I can be neglected. The current carriers are electrons. The average velocity of electrons in a semiconductor in the presence of an electric field only is $v = \mu E$, where μ is called mobility. If the magnetic field is also present, the electric field is no longer parallel to the current. This phenomenon is known as the Hall effect.

- a) Determine what the magnitude and the direction of the electric field in the bar is, to yield the current described above.
- b) Calculate the difference of the electric potential between the opposite points on the surfaces of the bar in the direction of the edge b .
- c) Find the analytic expression for the DC component of the electric potential difference in case b) if the current and the magnetic field are alternating (AC); $I = I_0 \sin \omega t$ and $B = B_0 \sin(\omega t + \delta)$.
- d) Design and explain an electric circuit which would make possible, by exploiting the result c), to measure the power consumption of an electric apparatus connected with the AC network.

Data: The electron mobility in InSb is $7.8 \text{ m}^2\text{T/Vs}$, the electron concentration in InSb is $2.5 \cdot 10^{22} \text{ m}^{-3}$, $I = 1.0 \text{ A}$, $B = 0.10 \text{ T}$, $b = 1.0 \text{ cm}$, $c = 1.0 \text{ mm}$, $e_0 = -1.6 \cdot 10^{-19} \text{ As}$.

Problem 3

In a space research project two schemes of launching a space probe out of the Solar system are discussed. The first scheme (i) is to launch the probe with a velocity large enough to escape from the Solar system directly. According to the second one (ii), the probe is to approach one of the outer planets, and with its help change its direction of motion and reach the velocity necessary to escape from the Solar system. Assume that the probe moves under the gravitational field of only the Sun or the planet, depending on whichever field is stronger at that point.

- a) Determine the minimum velocity and its direction relative to the Earth's motion that should be given to the probe on launching according to scheme (i).
- b) Suppose that the probe has been launched in the direction determined in a) but with another velocity. Determine the velocity of the probe when it crosses the orbit of Mars, i. e., its parallel and perpendicular components with respect to this orbit. Mars is not near the point of crossing, when crossing occurs.
- c) Let the probe enter the gravitational field of Mars. Find the minimum launching velocity from the Earth necessary for the probe to escape from the Solar system.

Hint: From the result a) you know the optimal magnitude and the direction of the velocity of the probe that is necessary to escape from the Solar system after leaving the gravitational field of Mars. (You do not have to worry about the precise position of Mars during the encounter.) Find the relation between this velocity and the velocity components before the probe enters the gravitational field of Mars; i. e., the components you determined in b). What about the conservation of energy of the probe?

- d) Estimate the maximum possible fractional saving of energy in scheme (ii) with respect to scheme (i). Notes: Assume that all the planets revolve round the Sun in circles, in the same direction and in the same plane. Neglect the air resistance, the rotation of the Earth around its axis as well as the energy used in escaping from the Earth's gravitational field.

Data: Velocity of the Earth round the Sun is 30 km/s, and the ratio of the distances of the Earth and Mars from the Sun is $2/3$.

1.2 Experimental competition

Exercise A

Follow the acceleration and the deceleration of a brass disk, driven by an AC electric motor. From the measured times of half turns, plot the angle, angular velocity and angular acceleration of the disk as functions of time. Determine the torque and power of the motor as functions of angular velocity.

Instrumentation

1. AC motor with switch and brass disk
2. Induction sensor
3. Multichannel stop-watch (computer)

Instruction

The induction sensor senses the iron pegs, mounted on the disk, when they are closer than 0.5 mm and sends a signal to the stop-watch. The stop-watch is programmed on a computer so that it registers the time at which the sensor senses the approaching peg and stores it in memory. You run the stop-watch by giving it simple numerical commands, i. e. pressing one of the following numbers:

5 - MEASURE.

The measurement does not start immediately. The stop-watch waits until you specify the number of measurements, that is, the number of successive detections of the pegs:

3 - 30 measurements

6 - 60 measurements

Either of these commands starts the measurement. When a measurement is completed, the computer displays the results in graphic form. The vertical axis represents the length of the interval between detection of the pegs and the horizontal axis is the number of the interval.

7 - display results in numeric form.

The first column is the number of times a peg has passed the detector, the second is the time elapsed from the beginning of the measurement and the third column is the length of the time interval between the detection of the two pegs.

In the case of 60 measurements:

8 - displays the first page of the table

2 - displays the second page of the table

4 - displays the results graphically.

A measurement can be interrupted before the prescribed number of measurements by pressing any key and giving the disk another half turn.

The motor runs on 25 V AC. You start it with a switch on the mounting base. It may sometimes be necessary to give the disk a light push or to tap the base plate to start the disk.

The total moment of inertia of all the rotating parts is: $(14.0 \pm 0.5) \cdot 10^{-6} \text{ kgm}^2$.

Exercise B

Locate the position of the centers and determine the orientations of a number of identical permanent magnets hidden in the black painted block. A diagram of one such magnet is given in Figure 1. The coordinates x , y and z should be measured from the red corner point, as indicated in Figure 2.

Determine the z component of the magnetic induction vector \vec{B} in the (x, y) plane at $z = 0$ by calibrating the measuring system beforehand.

Find the greatest magnetic induction B obtainable from the magnet supplied.

Instrumentation

1. Permanent magnet given is identical to the hidden magnets in the block.
2. Induction coil; 1400 turns, $R = 230 \Omega$

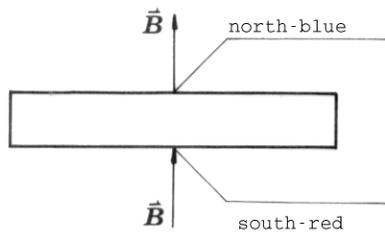


Fig. 1

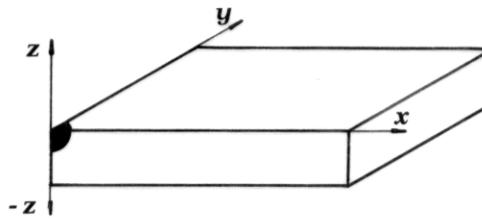


Fig. 2

3. Field generating coils, 8800 turns, $R = 990 \Omega$, 2 pieces
4. Black painted block with hidden magnets
5. Voltmeter (ranges 1 V, 3 V and 10 V recommended)
6. Electronic circuit (recommended supply voltage 24 V)
7. Ammeter
8. Variable resistor 3.3 k Ω
9. Variable stabilized power supply 0 - 25 V, with current limiter
10. Four connecting wires
11. Supporting plate with fixing holes
12. Rubber bands, multipurpose (e. g. for coil fixing)
13. Tooth picks
14. Ruler
15. Thread

Instructions

For the magnet-search only nondestructive methods are acceptable. The final report should include results, formulae, graphs and diagrams. The diagrams should be used instead of comments on the methods used wherever possible.

The proper use of the induced voltage measuring system is shown in Figure 3.

This device is capable of responding to the magnetic field. The peak voltage is proportional to the change of the magnetic flux through the coil.

The variable stabilized power supply is switched ON (1) or OFF (0) by the lower left pushbutton. By the (U) knob the output voltage is increased through the clockwise rotation. The recommended voltage is 24 V. Therefore switch the corresponding toggle switch to the 12 V - 25 V position. With this instrument either the output voltage U or the output current I is measured, with respect to the position of the corresponding toggle switch (V,A). However, to get the output voltage the upper right switch should be in the 'Vklop' position. By the knob (I) the output current is limited below the preset value. When rotated clockwise the power supply can provide 1.5 A at most.

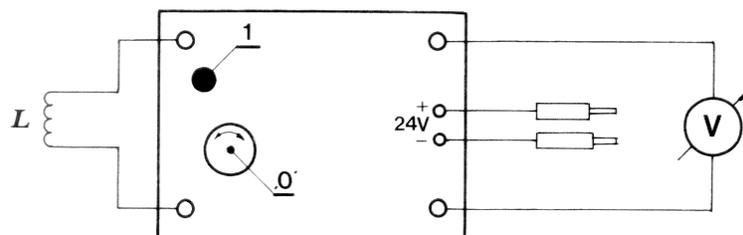


Fig. 3 '0' zero adjust dial, '1' push reset button

Note: permeability of empty space $\mu_0 = 1.2 \cdot 10^{-6}$ Vs/Am.

2 Solutions

2.1 Theoretical competition

Problem 1

a) Let the electrical signals supplied to rods 1 and 2 be $E_1 = E_0 \cos \omega t$ and $E_2 = E_0 \cos(\omega t + \delta)$, respectively. The condition for a maximum signal in direction ϑ_A (Fig. 4) is:

$$\frac{2\pi a}{\lambda} \sin \vartheta_A - \delta = 2\pi N$$

and the condition for a minimum signal in direction ϑ_B :

$$\frac{2\pi a}{\lambda} \sin \vartheta_B - \delta = 2\pi N' + \pi \quad (2p.)$$

where N and N' are arbitrary integers. In addition, $\vartheta_A - \vartheta_B = \varphi$, where

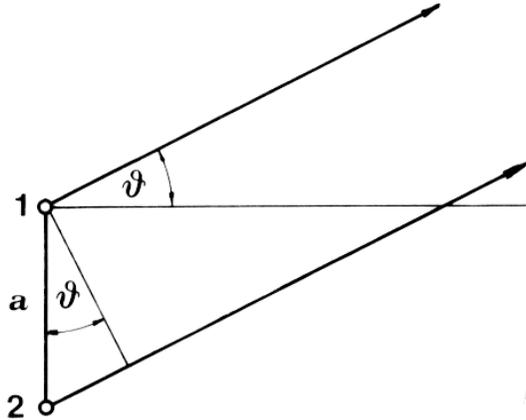


Fig. 4

φ is given. The problem can now be formulated as follows: Find the parameters a , ϑ_A , ϑ_B , δ , N , and N' satisfying the above equations such, that a is minimum.

We first eliminate δ by subtracting the second equation from the first one:

$$a \sin \vartheta_A - a \sin \vartheta_B = \lambda(N - N' - \frac{1}{2}).$$

Using the sine addition theorem and the relation $\vartheta_B = \vartheta_A - \varphi$:

$$2a \cos(\vartheta_A - \frac{1}{2}\varphi) \sin \frac{1}{2}\varphi = \lambda(N - N' - \frac{1}{2})$$

or

$$a = \frac{\lambda(N - N' - \frac{1}{2})}{2 \cos(\vartheta_A - \frac{1}{2}\varphi) \sin \frac{1}{2}\varphi}.$$

The minimum of a is obtained for the greatest possible value of the denominator, i. e.:

$$\cos(\vartheta_A - \frac{1}{2}\varphi) = 1, \quad \vartheta_A = \frac{1}{2}\varphi,$$

and the minimum value of the numerator, i. e.:

$$N - N' = 1.$$

The solution is therefore:

$$a = \frac{\lambda}{4 \sin \frac{1}{2}\varphi}, \quad \vartheta_A = \frac{1}{2}\varphi, \quad \vartheta_B = -\frac{1}{2}\varphi \quad \text{and} \quad \delta = \frac{1}{2}\pi - 2\pi N. \quad (6p.)$$

($N = 0$ can be assumed throughout without losing any physically relevant solution.)

b) The wavelength $\lambda = c/\nu = 11.1$ m, and the angle between directions A and B, $\varphi = 157^\circ - 72^\circ = 85^\circ$. The minimum distance between the rods is $a = 4.1$ m, while the direction of the symmetry line of the rods is $72^\circ + 42.5^\circ = 114.5^\circ$ measured from the north. (2 p.)

Problem 2

a) First the electron velocity is calculated from the current I:

$$I = jS = ne_0vbc, \quad v = \frac{I}{ne_0bc} = 25 \text{ m/s}.$$

The components of the electric field are obtained from the electron velocity. The component in the direction of the current is

$$E_{\parallel} = \frac{v}{\mu} = 3.2 \text{ V/m}. \quad (0.5\text{p.})$$

The component of the electric field in the direction b is equal to the Lorentz force on the electron divided by its charge:

$$E_{\perp} = vB = 2.5 \text{ V/m}. \quad (1\text{p.})$$

The magnitude of the electric field is

$$E = \sqrt{E_{\parallel}^2 + E_{\perp}^2} = 4.06 \text{ V/m}. \quad (0.5\text{p.})$$

while its direction is shown in Fig. 5 (Note that the electron velocity is in the opposite direction with respect to the current.) (1.5 p.)

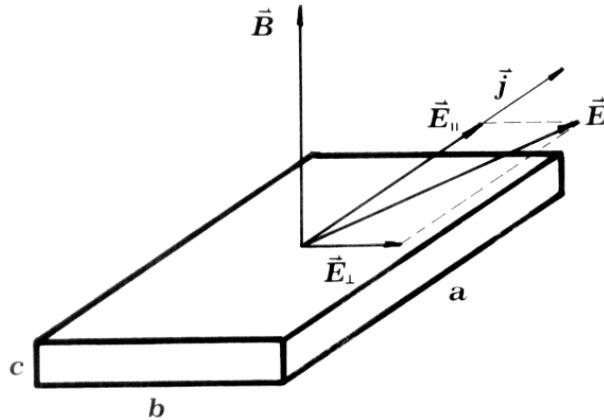


Fig. 5

b) The potential difference is

$$U_H = E_{\perp}b = 25 \text{ mV} \quad (1\text{p.})$$

c) The potential difference U_H is now time dependent:

$$U_H = \frac{IBb}{ne_0bc} = \frac{I_0B_0}{ne_0c} \sin \omega t \sin(\omega t + \delta).$$

The DC component of U_H is

$$\bar{U}_H = \frac{I_0B_0}{2ne_0c} \cos \delta. \quad (3p.)$$

d) A possible experimental setup is shown in Fig. 6

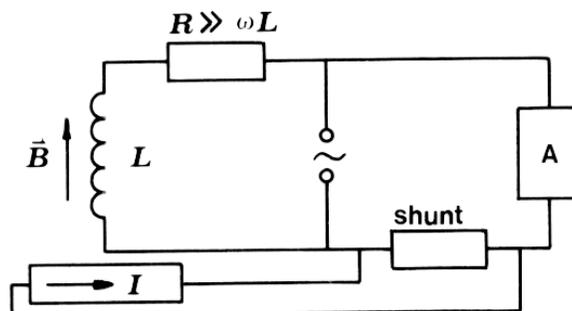


Fig. 6

(2 p.)

Problem 3

a) The necessary condition for the space-probe to escape from the Solar system is that the sum of its kinetic and potential energy in the Sun's gravitational field is larger than or equal to zero:

$$\frac{1}{2}mv_a^2 - \frac{GmM}{R_E} \geq 0,$$

where m is the mass of the probe, v_a its velocity relative to the Sun, M the mass of the Sun, R_E the distance of the Earth from the Sun and G the gravitational constant. Using the expression for the velocity of the Earth, $v_E = \sqrt{GM/R_E}$, we can eliminate G and M from the above condition:

$$v_a^2 \geq \frac{2GM}{R_E} = 2v_E^2. \quad (1p.)$$

Let v'_a be the velocity of launching relative to the Earth and ϑ the angle between \vec{v}_E and \vec{v}'_a (Fig. 7). Then from $\vec{v}_a = \vec{v}'_a + \vec{v}_E$ and $v_a^2 = 2v_E^2$ it

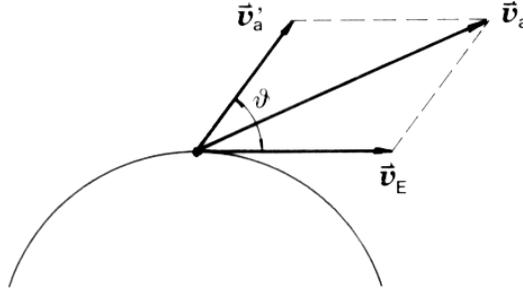


Fig. 7

follows:

$$v_a'^2 + 2v'_a v_E \cos \vartheta - v_E^2 = 0$$

and

$$v'_a = v_E \left[-\cos \vartheta + \sqrt{1 + \cos^2 \vartheta} \right].$$

The minimum velocity is obtained for $\vartheta = 0$:

$$v'_a = v_E(\sqrt{2} - 1) = 12.3 \text{ km/s}. \quad (1p.)$$

b) Let v'_b and v_b be the velocities of launching the probe in the Earth's and Sun's system of reference respectively. For the solution (a), $v_b = v'_b + v_E$. From the conservation of angular momentum of the probe:

$$mv_b R_E = mv_{\parallel} R_M \quad (1p.)$$

and the conservation of energy:

$$\frac{1}{2}mv_b^2 - \frac{GmM}{R_E} = \frac{1}{2}m(v_{\parallel}^2 + v_{\perp}^2) - \frac{GmM}{R_M} \quad (1p.)$$

we get for the, parallel component of the velocity (Fig. 8):

$$v_{\parallel} = (v'_b + v_E)k,$$

and for the perpendicular component:

$$v_{\perp} = \sqrt{(v'_b + v_E)^2(1 - k^2) - 2v_E^2(1 - k)}. \quad (1p.)$$

where $k = R_E/R_M$.

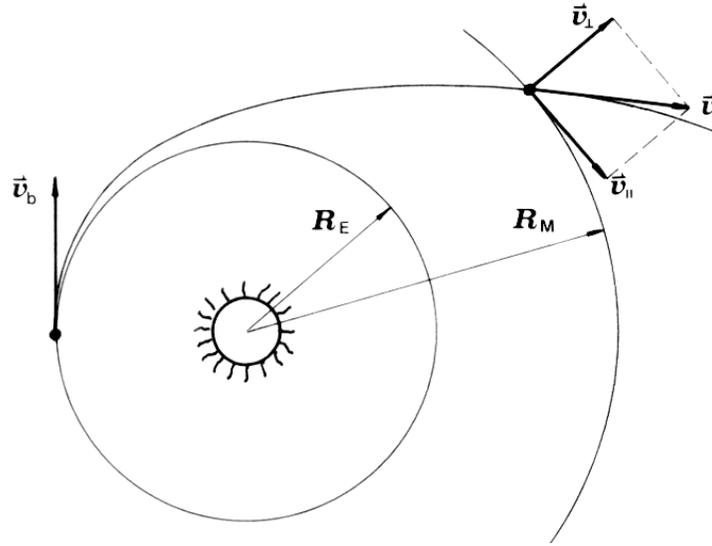


Fig. 8

c) The minimum velocity of the probe in the Mars' system of reference to escape from the Solar system, is $v_s'' = v_M(\sqrt{2} - 1)$, in the direction parallel to the Mars orbit (v_M is the Mars velocity around the Sun). The role of Mars is therefore to change the velocity of the probe so that it leaves its gravitational field with this velocity.

(1 p.)

In the Mars' system, the energy of the probe is conserved. That is, however, not true in the Sun's system in which this encounter can be considered as an elastic collision between Mars and the probe. The velocity of the probe before it enters the gravitational field of Mars is therefore, in

the Mars' system, equal to the velocity with which the probe leaves its gravitational field. The components of the former velocity are $v''_{\perp} = v_{\perp}$ and $v''_{\parallel} = v_{\parallel} - v_M$, hence:

$$v'' = \sqrt{v''_{\parallel}{}^2 + v''_{\perp}{}^2} = \sqrt{v_{\perp}^2 + (v_{\parallel} - v_M)^2} = v'_s. \quad (1p.)$$

Using the expressions for v_{\perp} and v_{\parallel} from (b), we can now find the relation between the launching velocity from the Earth, v'_b , and the velocity v'_s , $v'_s = v_M(\sqrt{2} - 1)$:

$$(v'_b + v_E)^2(1 - k^2) - 2v_E^2(1 - k) + (v'_b + v_E)^2k^2 - 2v_M(v'_b + v_E)k = v_M^2(2 - 2\sqrt{2}).$$

The velocity of Mars round the Sun is $v_M = \sqrt{GM/R_M} = \sqrt{k} v_E$, and the equation for v'_b takes the form:

$$(v'_b + v_E)^2 - 2\sqrt{k}^3 v_E(v'_b + v_E) + (2\sqrt{2}k - 2)v_E^2 = 0. \quad (1p.)$$

The physically relevant solution is:

$$v'_b = v_E \left[\sqrt{k}^3 - 1 + \sqrt{k^3 + 2 - 2\sqrt{2}k} \right] = 5.5 \text{ km/s}. \quad (1p.)$$

d) The fractional saving of energy is:

$$\frac{W_a - W_b}{W_a} = \frac{v'_a{}^2 - v'_b{}^2}{v'_a{}^2} = 0.80,$$

where W_a and W_b are the energies of launching in scheme (i) and in scheme (ii), respectively. (1 p.)

2.2 Experimental competition

Exercise A

The plot of the angle as a function of time for a typical measurement of the acceleration of the disk is shown in Fig. 9.

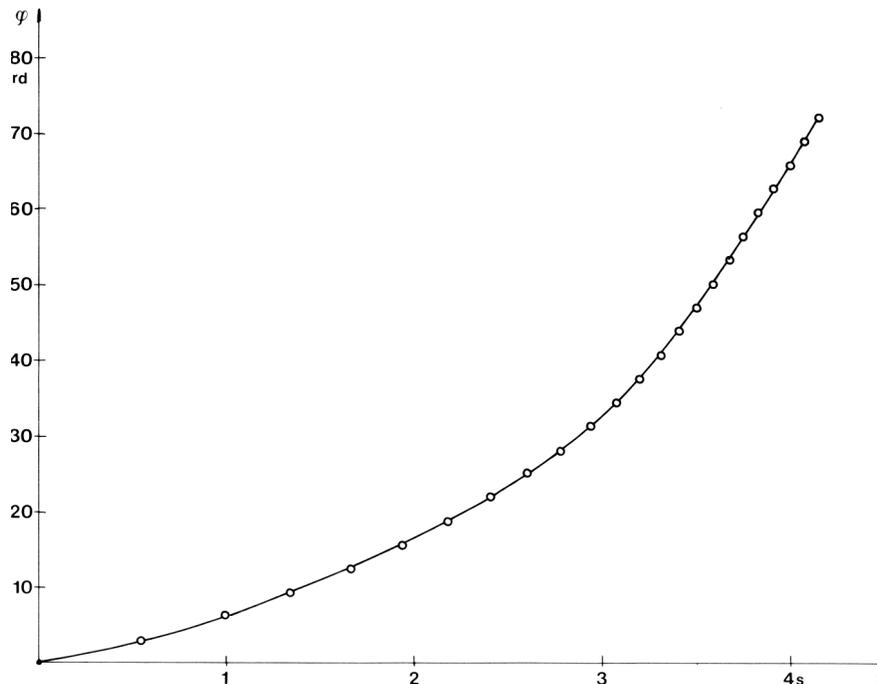


Fig. 9 Angle vs. time

The angular velocity is calculated using the formula:

$$\omega_i(t'_i) = \frac{\pi}{(t_{i+1} - t_i)}$$

and corresponds to the time in the middle of the interval (t_i, t_{i+1}) : $t'_i = \frac{1}{2}(t_{i+1} + t_i)$. The calculated values are displayed in Table 1 and plotted in Fig. 10.

Observing the time intervals of half turns when the constant angular velocity is reached, one can conclude that the iron pegs are not positioned perfectly symmetrically. This systematic error can be neglected in the calculation of angular velocity, but not in the calculation of angular acceleration. To avoid this error we use the time intervals of full turns:

$$\alpha_i(t''_i) = \frac{\Delta\omega_i}{\Delta t_i},$$

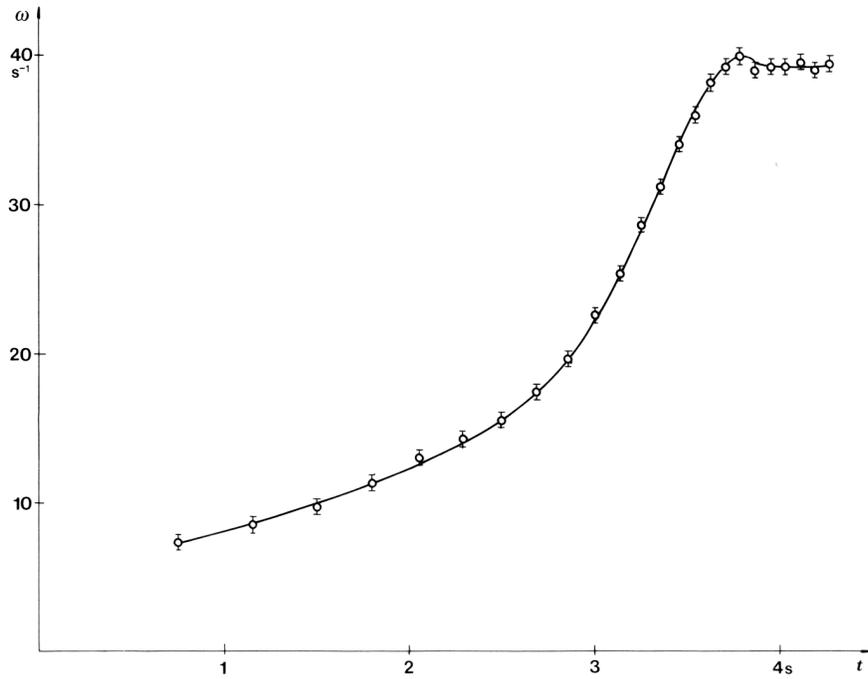


Fig. 10 Angular velocity vs. time

where $\Delta t_i = t_{2i+2} - t_{2i}$,

$$\Delta\omega_i = \frac{2\pi}{(t_{2i+3} - t_{2i+1})} - \frac{2\pi}{(t_{2i+1} - t_{2i-1})}$$

and $t_i'' = t_{2i+1}'$.

The angular acceleration as a function of time is plotted in Fig. 11.

The torque, M , and the power, P , necessary to drive the disk (net torque and net power), are calculated using the relation:

$$M(t) = I\alpha(t)$$

and

$$P(t) = M(t)\omega(t)$$

where the moment of inertia, $I = (14.0 \pm 0.5) \cdot 10^{-6} \text{ kgm}^2$, is given. The corresponding angular velocity is determined from the plot in Fig. 10 by interpolation. This plot is used also to find the torque and the power as functions of angular velocity (Fig. 12 and 13).

i	t ms	δt ms	φ rd	t' ms	ω s^{-1}	α s^{-2}
1	0.0		0.0			
2	543.9	543.9	3.14	272.0	5.78	
3	973.5	429.6	6.28	758.7	7.31	3.38
4	1339.0	365.5	9.42	1156.3	8.60	
5	1660.8	327.8	12.57	1499.9	9.76	5.04
6	1936.3	275.5	15.71	1798.6	11.40	
7	2177.8	241.5	18.85	2057.1	13.01	5.96
8	2396.6	218.8	21.99	2287.2	14.36	
9	2599.6	203.0	25.73	2498.1	15.48	9.40
10	2779.5	179.9	28.27	2689.6	17.46	
11	2939.3	159.8	31.42	2859.4	19.66	18.22
12	3078.0	138.7	34.56	3008.6	22.65	
13	3201.8	123.8	37.70	3139.9	25.38	25.46
14	3311.4	109.6	40.84	3256.6	28.66	
15	3472.1	100.7	43.98	3361.8	31.20	26.89
16	3504.2	92.1	47.12	3458.2	34.11	
17	3591.3	87.1	50.27	3547.8	36.07	21.72
18	3673.4	82.1	53.41	3632.4	38.27	
19	3753.5	80.1	56.55	3713.5	39.22	4.76
20	3832.7	78.6	59.69	3792.8	39.97	
21	3912.6	80.5	62.83	3872.4	39.03	-1.69
22	3992.7	80.1	65.97	3952.7	39.22	
23	4072.8	80.1	69.12	4032.8	39.22	0.77
24	4152.0	79.2	72.26	4112.4	39.67	
25	4232.5	80.5	75.40	4192.3	39.03	-0.15
26	4312.3	79.7	78.54	4272.4	39.42	

Table 1

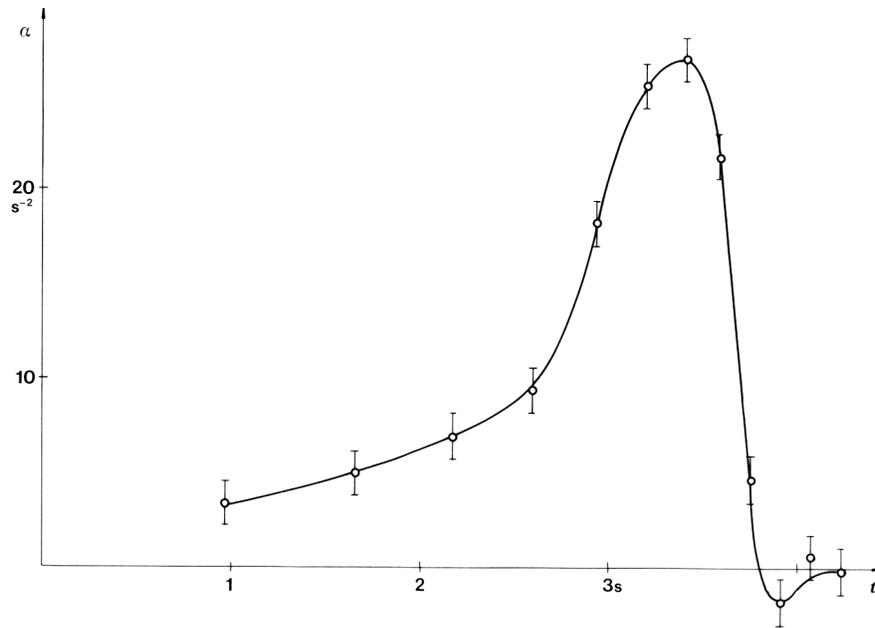


Fig. 11 Angular acceleration vs. time

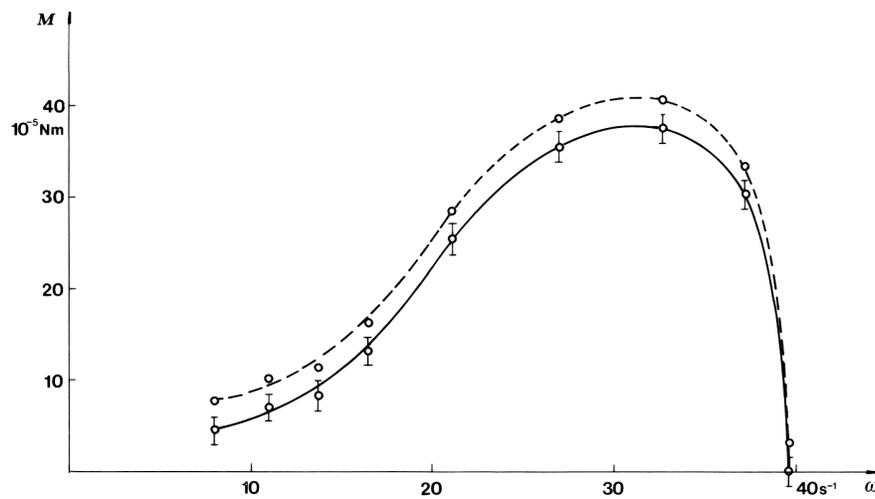


Fig. 12 Net torque (full line) and total torque (dashed line) vs. angular velocity

To find the total torque and the power of the motor, the torque and the power losses due to the friction forces have to be determined and added to the corresponding values of net torque and power. By measuring the angular velocity during the deceleration of the disk after the motor has

been switched off (Fig. 14), we can determine the torque of friction which is approximately constant and is equal to $M' = (3.1 \pm 0.3) \cdot 10^{-5}$ Nm.

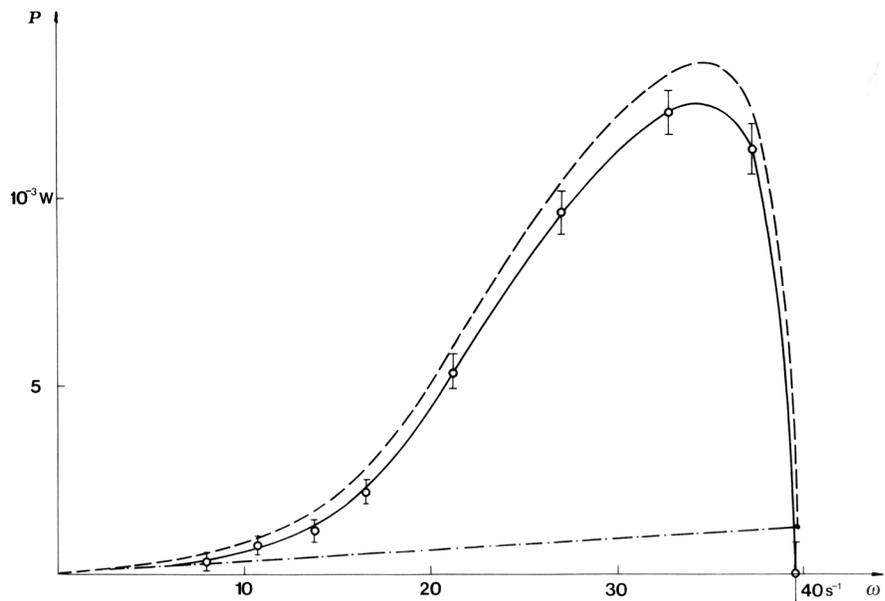


Fig. 13 Net power (full line), power losses (dashed and dotted line) and total power (dashed line) vs. angular velocity

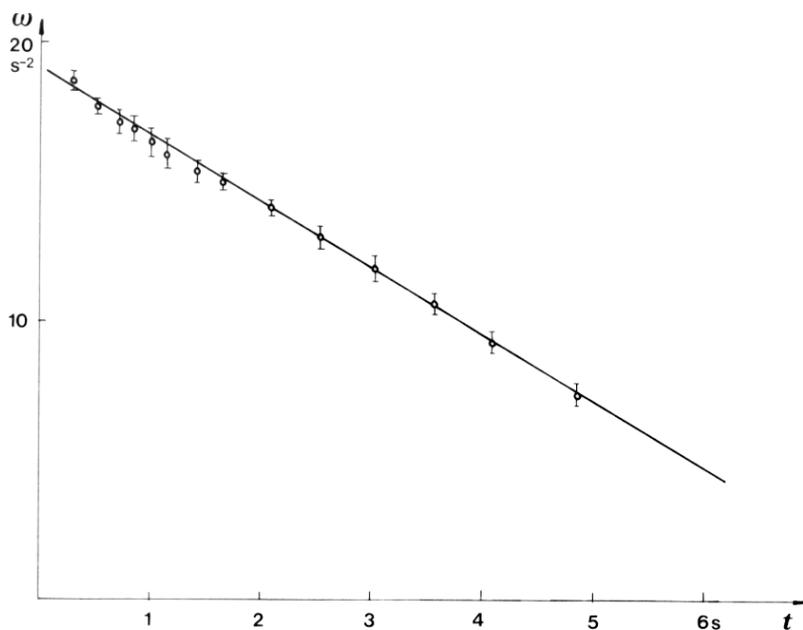


Fig. 14 Angular velocity vs. time during deceleration

The total torque and the total power are shown in Fig. 12 and 13.

Marking scheme

- a) Determination of errors 1 p.
- b) Plot of angle vs. time 1 p.
- c) Plot of angular velocity and acceleration 3 p.
- d) Correct times for angular velocity 1 p.
- e) Plot of net torque vs. angular velocity 2 p. (Plot of torque vs. time only, 1 p.)
- f) Plot of net power vs. angular velocity 1 p.
- g) Determination of friction 1 p.

Exercise B

Two permanent magnets having the shape of rectangular parallelepipeds with sides 50 mm, 20 mm and 8 mm are hidden in a block of polystyrene foam with dimension 50 cm, 31 cm and 4.0 cm. Their sides are parallel to the sides of the block. One of the hidden magnets (A) is positioned so that its \vec{B} (Fig. 15) points in z direction and the other (B) with its \vec{B} in x or y direction (Fig. 15).

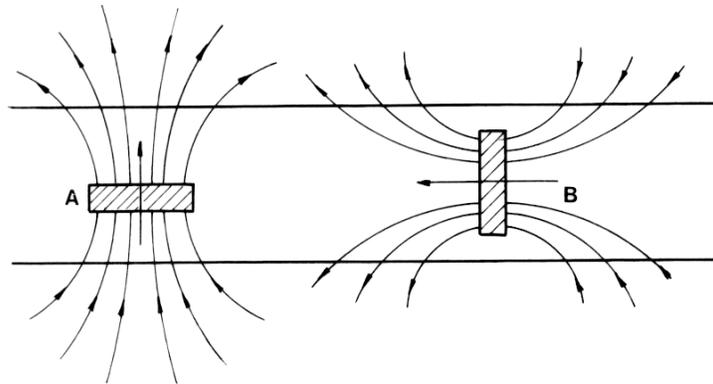


Fig. 15 A typical implementation of the magnets in the block

The positions and the orientations of the magnets should be determined on the basis of observations of forces acting on the extra magnet. The best way to do this is to hang the extra magnet on the thread and move it above the surface to be explored. Three areas of strong forces are revealed when the extra magnet is in the horizontal position i. e. its \vec{B} is parallel to z axis, suggesting that three magnets are hidden. Two of these areas producing an attractive force in position P (Fig. 16) and a repulsive force in position R are closely together.

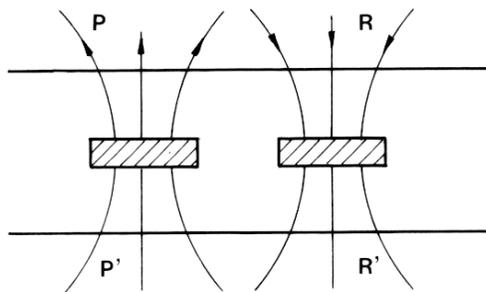


Fig: 16 Two 'ghost' magnets appearing in the place of magnet B

However, by inspecting the situation on the other side of the block, again an attractive force in area P' is found, and a repulsive one in area R'. This is in the contradiction with the supposed magnets layout in Fig. 16 but corresponds to the force distribution of magnet B in Fig. 15.

To determine the z position of the hidden magnets one has to measure the z component of \vec{B} on the surface of the block and compare it to the measurement of B_z of the extra magnet as a function of distance from its center (Fig. 18). To achieve this the induction coil of the measuring system is removed from the point in which the magnetic field is measured to a distance in which the magnetic field is practically zero, and the peak voltage is measured.

In order to make the absolute calibration of the measuring system, the response of the system to the known magnetic field should be measured. The best defined magnetic field is produced in the gap between two field generating coils. The experimental layout is displayed in Fig. 17.

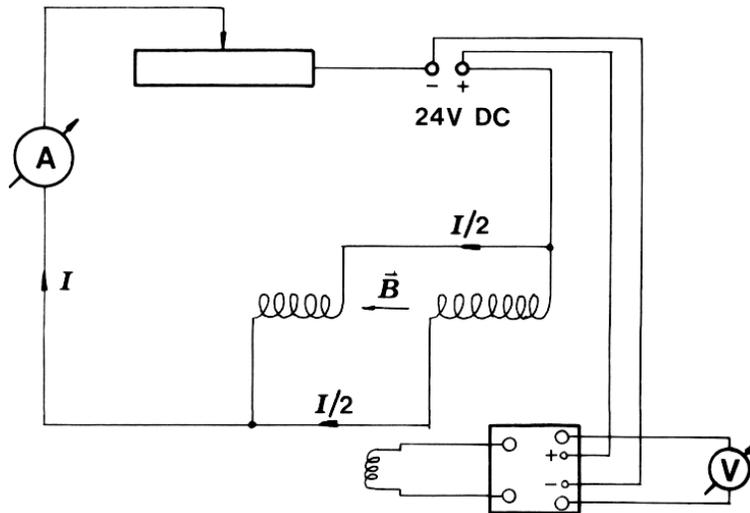


Fig. 17 Calibration of the measuring system

The magnetic induction in the gap between the field generating coils is calculated using the formula:

$$B = \frac{\mu_0 N I}{(2l + d)}$$

Here N is the number of the turns of one of the coils, l its length, d the width of the gap, and I the current through the ammeter. The peak voltage, U , is measured when the induction coil is removed from the gap.

Plotting the magnetic induction B as a function of peak voltage, we can determine the sensitivity of our measuring system:

$$\frac{B}{U} = 0.020 \text{ T/V.}$$

(More precise calculation of the magnetic field in the gap, which is beyond the scope of the exercise, shows that the true value is only 60 % of the value calculated above.)

The greatest value of B is 0.21 T.

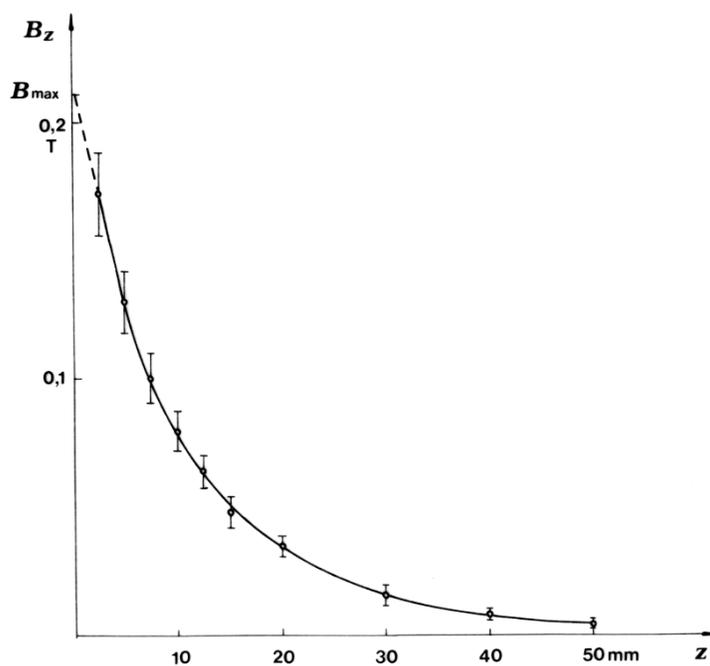


Fig. 18 Magnetic induction vs. distance

Marking scheme:

- a) determination of x, y position of magnets (± 1 cm) 1 p.
- b) determination of the orientations 1 p.
- c) depth of magnets (± 4 mm) 2 p.
- d) calibration (± 50 %) 3 p.
- e) mapping of the magnetic field 2 p.
- f) determination of B_{\max} (± 50 %) 1 p.

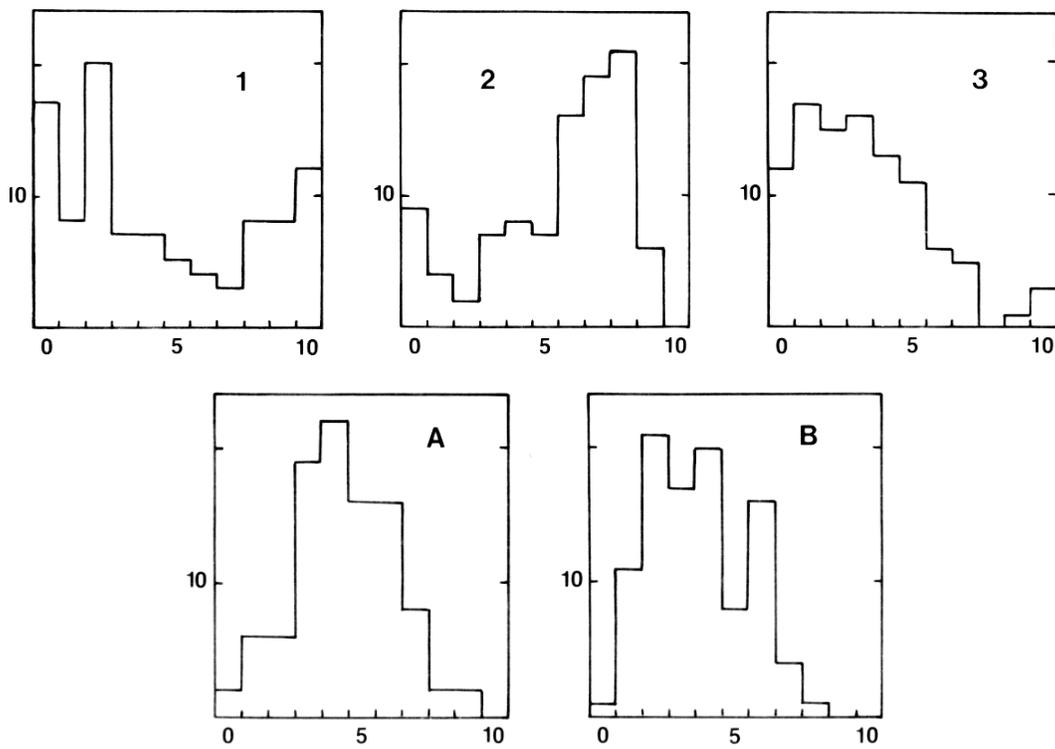


Fig. 19 Distribution of marks for the theoretical (1,2,3) and the experimental exercises. The highest mark for each exercise is 10 points.