

MARKING SCHEME FOR ANSWERS TO THE THEORETICAL QUESTION II

Part	MARKING SCHEME - THE THEORETICAL QUESTION II - DIFFERENT KIND OF OSCILLATION	Total Scores
II.a.	For: $\begin{cases} C = C_1 + C_2 \\ L = \frac{L_1 L_2}{L_1 + L_2} \end{cases} \quad 0.2p$ the impedance Z of the circuit $Z = \bar{Z} = \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(C \cdot \omega - \frac{1}{L \cdot \omega}\right)^2}} = \frac{1}{Y} \quad 0.2p$ $i(t) = I \cdot \sqrt{2} \cdot \sin(\omega \cdot t) \quad 0.2p$ $u(t) = U \cdot \sqrt{2} \cdot \sin(\omega \cdot t + \varphi) \quad 0.2p$ $P = \frac{U^2}{R} = \frac{Z^2 \cdot I^2}{R} \quad 0.1p$ the maximal active power is realized for the maximum value of the impedance that is the minimal value of the admittance $Y_{\min} = \frac{1}{R} \quad 0.2p$ $P_m = R \cdot I^2 \quad 0.2p$ $f_m = \frac{1}{2\pi \cdot \sqrt{C \cdot L}} \quad 0.2p$ $P = \frac{1}{2} P_m \quad 0.1p$ $\frac{Z^2 \cdot I^2}{R} = \frac{1}{2} R \cdot I^2 \quad 0.1p$ $\omega^2 \pm \frac{1}{R \cdot C} \omega - \frac{1}{L \cdot C} = 0 \quad 0.2p$ the pulsation of the current ensuring an active power at half of the maximum power $\begin{cases} f_+ = \frac{1}{2\pi} \left(\frac{1}{2} \sqrt{\left(\frac{1}{R \cdot C}\right)^2 + \frac{4}{L \cdot C}} + \frac{1}{2R \cdot C} \right) \\ f_- = \frac{1}{2\pi} \left(\frac{1}{2} \sqrt{\left(\frac{1}{R \cdot C}\right)^2 + \frac{4}{L \cdot C}} - \frac{1}{2R \cdot C} \right) \end{cases} \quad 0.2p$ the bandwidth of the circuit $\Delta f = f_+ - f_- = \frac{1}{2\pi} \frac{1}{R \cdot C} \quad 0.1p$ $\begin{cases} \frac{f_m}{\Delta f} = R \sqrt{\frac{C}{L}} \\ \frac{f_m}{\Delta f} = R \sqrt{\frac{(C_1 + C_2) \cdot (L_1 + L_2)}{L_1 \cdot L_2}} \end{cases} \quad 0.2p$ final result: $\frac{f_m}{\Delta f} = 150 \quad 0.1p$	2.5 points

II.b.		2.2 points
	For: $u(t) - L_1 \frac{di_1}{dt} = 0 \quad 0.1p$ $u(t) - L_2 \frac{di_2}{dt} = 0 \quad 0.1p$ $q_1 = C_1 \cdot u \quad 0.1p$ $\frac{dq_1}{dt} = -i_3 \quad 0.1p$ $\frac{dq_1}{dt} = C_1 \cdot \frac{du}{dt} \quad 0.1p$ $i_3 = -C_1 \cdot \frac{du}{dt} \quad 0.1p$ $i_4 = -C_2 \cdot \frac{du}{dt} \quad 0.1p$ $\begin{cases} \frac{di_1}{dt} = \frac{u}{L_1} \\ \frac{di_2}{dt} = \frac{u}{L_2} \end{cases} \quad 0.2p$ $\begin{cases} \frac{di_3}{dt} = -C_1 \frac{d^2u}{dt^2} \\ \frac{di_4}{dt} = -C_2 \frac{d^2u}{dt^2} \end{cases} \quad 0.2p$ the Kirchhoff rule of the currents for the point A $i_1 + i_5 = i_3$ 0.1p the Kirchhoff rule of the currents for the point B $i_4 + i_5 = i_2$ 0.1p $i_1 - i_3 = i_4 - i_2 \quad 0.1p$ $\frac{di_1}{dt} - \frac{di_3}{dt} = \frac{di_4}{dt} - \frac{di_2}{dt} \quad 0.1p$ $\begin{cases} -\frac{u}{L_1} - \frac{u}{L_2} = C_1 \frac{d^2u}{dt^2} + C_2 \frac{d^2u}{dt^2} \\ -u \cdot \left(\frac{1}{L_1} + \frac{1}{L_2} \right) = \frac{d^2u}{dt^2} \cdot (C_1 + C_2) \end{cases} \quad 0.2p$ $\begin{cases} -\frac{u}{L} = \frac{d^2u}{dt^2} \cdot C \\ \ddot{u} + \frac{1}{LC} u = 0 \end{cases} \quad 0.2p$ $\omega = \frac{1}{\sqrt{L \cdot C}} \quad 0.2p$	

	final result: $\omega = 10^5 \text{ rad} \cdot \text{s}^{-1}$; $f = \frac{10^5}{2\pi} \text{ Hz}$	0.1p	
II.c.	For: $u(t) = A \cdot \sin(\omega \cdot t + \delta)$ $\begin{cases} i_3 = -C_1 \frac{d}{dt} (A \cdot \sin(\omega \cdot t + \delta)) = -C_1 \cdot A \cdot \omega \cdot \cos(\omega \cdot t + \delta) \\ i_4 = -C_2 \frac{d}{dt} (A \cdot \sin(\omega \cdot t + \delta)) = -C_2 \cdot A \cdot \omega \cdot \cos(\omega \cdot t + \delta) \end{cases}$ $\begin{cases} \frac{di_1}{dt} = \frac{u}{L_1} = \frac{1}{L_1} \cdot A \cdot \sin(\omega \cdot t + \delta) \\ \frac{di_2}{dt} = \frac{u}{L_2} = \frac{1}{L_2} \cdot A \cdot \sin(\omega \cdot t + \delta) \end{cases}$ $\begin{cases} i_1 = \frac{1}{L_1 \cdot \omega} \cdot A \cdot \cos(\omega \cdot t + \delta) + M \\ i_2 = \frac{1}{L_2 \cdot \omega} \cdot A \cdot \cos(\omega \cdot t + \delta) + N \end{cases}$ Observation: in the expression above, A, M, N and δ are constants that must be determined using initially conditions ($u_0 = 40 \text{ V}$, $i_{01} = 0,1 \text{ A}$, $i_{02} = 0,2 \text{ A}$) $\begin{cases} u(0) = A \cdot \sin(\delta) = u_0 \\ \sin(\delta) = \frac{u_0}{A} \end{cases}$ $\begin{cases} i_{01} = \frac{1}{L_1 \cdot \omega} \cdot A \cdot \cos(\delta) + M \\ i_{02} = \frac{1}{L_2 \cdot \omega} \cdot A \cdot \cos(\delta) + N \end{cases}$ $\begin{cases} i_1 - i_3 = i_4 - i_2 \\ \frac{1}{L_1 \cdot \omega} \cdot A \cdot \cos(\omega \cdot t + \delta) + M + C_1 \cdot A \cdot \omega \cdot \cos(\omega \cdot t + \delta) = \\ -C_2 \cdot A \cdot \omega \cdot \cos(\omega \cdot t + \delta) - \frac{1}{L_2 \cdot \omega} \cdot A \cdot \cos(\omega \cdot t + \delta) - N \end{cases}$ Observation: an identity as $A \cdot \cos \alpha + B \equiv C \cdot \cos \alpha + D$ is valuable for any value of the argument α only if $A = C$ and $B = D$ $\begin{cases} M + N = 0 \\ A \cdot \omega \cdot (C_1 + C_2) = -\frac{A}{\omega} \cdot \left(\frac{1}{L_1} + \frac{1}{L_2} \right) \end{cases}$ $\begin{cases} i_{01} + i_{02} = A \cdot \cos(\delta) \cdot \frac{1}{\omega} \cdot \left(\frac{1}{L_1} + \frac{1}{L_2} \right) \\ \cos \delta = \frac{i_{01} + i_{02}}{A \cdot \frac{1}{\omega} \cdot \left(\frac{1}{L_1} + \frac{1}{L_2} \right)} \\ \cos \delta = \frac{(i_{01} + i_{02}) \cdot L \cdot \omega}{A} \end{cases}$	0.1p 0.1p 0.2p 0.2p 0.2p 0.1p 0.2p 0.2p 0.1p 0.2p	3.0 points

	$\begin{cases} (\cos(\delta))^2 + (\sin(\delta))^2 = 1 \\ \left(\frac{u_0}{A}\right)^2 + \left(\frac{(i_{01} + i_{02}) \cdot L \cdot \omega}{A}\right)^2 = 1 \\ A = \sqrt{(u_0)^2 + ((i_{01} + i_{02}) \cdot L \cdot \omega)^2} \end{cases}$	0.1p
	the numerical value of the electrical tension on the jacks of the circuit $A = 40\sqrt{26} \text{ V}$	0.1p
	$\begin{cases} \sin(\delta) = \frac{u_0}{A} \\ \sin(\delta) = \frac{1}{\sqrt{26}} \end{cases}$	0.1p
	$\begin{cases} M = i_{01} - \frac{1}{L_1 \cdot \omega} \cdot A \cdot \cos(\delta) \\ N = i_{02} - \frac{1}{L_2 \cdot \omega} \cdot A \cdot \cos(\delta) \end{cases}$	0.1p
	$\begin{cases} M = -0,1 \text{ A} \\ N = 0,1 \text{ A} \end{cases}$	0.1p
	$\begin{cases} i_1 = \left(\frac{4\sqrt{26}}{100} \cdot \cos(10^5 \cdot t + \arctg(1/5)) - 0,1 \right) A = \tilde{i}_1 - I_0 \\ i_2 = \left(\frac{2\sqrt{26}}{100} \cdot \cos(10^5 \cdot t + \arctg(1/5)) + 0,1 \right) A = \tilde{i}_2 + I_0 \end{cases}$	0.2p
	Observation: the currents through the coils are the superposition of sinusoidal currents having different amplitudes and a direct current passing only through the coils. the direct current $I_0 = 0,1 \text{ A}$	0.1p
	the alternative currents through the coils $\begin{cases} \tilde{i}_1 = \left(\frac{4\sqrt{26}}{100} \cdot \cos(10^5 \cdot t + \arctg(1/5)) \right) A \\ \tilde{i}_2 = \left(\frac{2\sqrt{26}}{100} \cdot \cos(10^5 \cdot t + \arctg(1/5)) \right) A \end{cases}$	0.2p
	the currents through the capacitors $\begin{cases} i_3 = \left(-\frac{4\sqrt{26}}{100} \cos(10^5 \cdot t + \arctg(1/5)) \right) A \\ i_4 = \left(-\frac{2\sqrt{26}}{100} \cos(10^5 \cdot t + \arctg(1/5)) \right) A \end{cases}$	0.2p
	$i_5 = i_3 - i_1$	0.1p
	final result: $i_5 = \left[-\frac{8\sqrt{26}}{100} \cos\left(10^5 t + \arctg \frac{1}{5}\right) + 0,1 \right] A$	0.2p
II.d.	For: the amplitude of the current through the inductance L_1 $\max(\tilde{i}_1) = \max\left(\frac{4\sqrt{26}}{100} \cdot \cos(10^5 \cdot t + \arctg(1/5)) A\right)$ final result: $\max(\tilde{i}_1) \approx 0,2 \text{ A}$	0.3 points 0.2p 0.1p
Total score theoretical question III		8.0 points

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